Journal of Russian & East European Psychology

Matusov, E. (2017). Nikolai N. Konstantinov's authorial math pedagogy for people with wings: Special issue. *Journal of Russian & East European Psychology, 54*(1), 1-117, doi: https://doi.org/10.1080/10610405.2017.1352391
See Table of corrections on the page 118

ISSN: 1061-0405 (Print) 1558-0415 (Online) Journal homepage: http://www.tandfonline.com/loi/mrpo20

Nikolai N. Konstantinov's Authorial Math Pedagogy for People with Wings

Eugene Matusov

Journal of Russian & East European Psychology, vol. 54, no. 1, 2017, pp. 1–117.

ISSN: 1061-0405 (print)/ISSN 1558-0415 (online)
DOI: https://doi.org/10.1080/10610405.2017.1352391

EUGENE MATUSOV

Nikolai N. Konstantinov's Authorial Math Pedagogy for People with Wings

This special issue is dedicated to an innovative pedagogy by Soviet-Russian math educator Nikalai Nikolaevich Konstantinov. Diverse and at times contradictory interviews with Konstantinov, math teachers involved in his pedagogy, and former students, available sources in Russian and English, and my own memoirs as a former student of Konstantinov, I tried to reconstruct, define, analyze, evaluate, and problematize his innovative pedagogy. Konstantinov himself defined his innovative pedagogy as promoting "people with wings" – promoting initiative, creativity, ownership, critical thinking, and self-realization among students in math and other areas. In math instruction, Konstantinov focuses on providing students with choices of math problems, interrogating students' math solutions, and offering guidance in a direct response to the questions and difficulties that the students experience in their particular math problems. I demonstrate his pedagogical approach is integrative aiming not only at math. Emerging tensions between students' curricular choice and teacher's imposition, educational elitism and social equality, and teacher's authorial freedom and Konstantinov's support are discussed.

Introduction

Since 2011, I teach my students—undergraduate, future educators, and graduate, future researchers of education—using the "Open

Curriculum" pedagogical regime. The Open Curriculum involves a long list of possible curricular topics for the course that I develop based on my own professional knowledge of the subject and my own paradigmatic inclinations and taste. I also search on the Internet for syllabi of my colleagues around the world teaching this subject and add their topics to my list that I call "Curricular Map." Students can also amend the Curricular Map by adding new curricular topics relevant to the class. In the Curricular Map, I accompany each topic with a "teaser," a short paragraph provoking students' interest in the topic. I invite my students to add their own topics (and teasers) on the Curricular Map via our class web. At the end of each class, my students vote twice on the next topic of the class. The first time they vote individually as many times as they want. The second time they vote only on one of the several topics that got the most votes in the first selection. Prior to the second vote, students try to convince their peers to vote on their choice. Sometimes a class chooses two or even three topics to study. Sometimes they prefer to stay on the same topic next class. Sometimes the class splits into smaller groups with leaders to study their own topic. In my observation, the Open Curriculum promotes students' learning activism and ownership for their own education (Matusov, 2015b; Matusov and Marjanovic-Shane, 2017). As a teacher, I see myself as a guide to a foreign country for my students, who develop their own interests and inclinations in it.

It happened that for the last three semesters, starting from fall 2015, I am getting several math education doctoral graduate students in my classes. On several occasions, they challenged me by claiming that Open Curriculum might be good for teaching social science classes, such as our classes on educational research and theory, but might be bad for disciplines such as math. In their view, math requires a linear strict progression of the curriculum, and, thus, the traditional Closed Curriculum, where the teacher, not the students, unilaterally decides what the students must study each class. I have never taught any math subject using the Open Curriculum and initially I thought that it was an interesting empirical question. But then…

I suddenly remember that, actually, I experienced a version of the Open Curriculum in my Moscow Math High School No. 91 in the Soviet Union in 1974 – 1977. In that school, we studied "math analysis" (calculus and many other math subjects) with leaflets that involved short definitions of math concepts and numerous interesting math problems (see Appendix). We could freely choose the problems to solve in any order and, thus, build our own individual curriculum. We also autonomously, self-directedly, explored foreign lands of diverse math units with or without the help of our teachers. My math graduate students asked me many questions about the math innovative pedagogical practices that I experienced in the USSR but I could not provide many answers. I knew that the author of this math pedagogy was Nikolai[1] Nikolaevich Konstantinov but that was basically all I knew. Of course, I had my memories, observations, and reflections on my experiences but they were partial and I was not sure that they were correct. Then I was thinking, "Eugene, you are an educational researcher. Why don't you interview Nikolai Nikolaevich and other participants and explore literature in Russian about him and his math pedagogy?! The interest in his innovative pedagogy is here!"

My first attempt, fall 2015, to contact Konstantinov failed. I contacted the administration of the math school and the Independent University of Moscow that Nikolai Konstantinov founded and worked in but received no reply. In fall 2016, my math education doctoral students started asking me again about Konstantinov. This time I was luckier. I found interviews with Nikolai Nikolaevich in Russian on the Internet and contacted several journalists who interviewed Konstantinov. Additionally, I found a site regarding the celebration of his 80th birthday in 2012 and wrote to its organizers. Finally, I got an email from a Russian prospective graduate student who wanted to apply to my university as a math educator. He did not know Konstantinov but found people who did. In a few days, I got Konstantinov's home phone number and his archive.

Methodology

I decided to interview Nikolai Nikolaevich. He could not use Skype or email and we agreed to do it by phone. I used Skype phone to record our conversation. He wanted me to start our interview immediately. I sensed some tension in his voice as, I thought, he was not sure he could trust me. He told me that he heard my name recently from his colleagues as I was searching for his contact. In addition, the fact that I was an alumnus of the Moscow Math School No. 91 was reassuring for him. He asked me what my interest was. When I read my prepared questions, Nikolai Nikolaevich was apparently overwhelmed. Then, I improvised another approach. I told him, "Why don't I tell you my memories, observations, experiences, and reflections on my math circle and my math school and you would check if they are accurate or not?" He liked the idea and our conversations started. Thus, I shifted from a genre of interview of me asking questions and him replying, to a genre of conversation of a free-flow reflective chat between us. Both he and I were active participants of his innovative pedagogical practice. I shared my memories, observations, reflections, asked him for clarification and verification, and inquired about his experiences and pedagogical ideas. Nikolai Nikolaevich also asked me questions and shared his experiences and ideas. At the end of an hour, I felt that Nikolai Nikolaevich, at age 84, got tired, although he still sounded very excited and animated. I suggested to postpone our conversation until next week. I asked him if he had more time to talk and he replied, "I have absolutely no time. I find it interesting to talk with you, but it would be untrue to say that I have a lot of time." We had two more phone interviews, which amounted to a total of about 4.5 hours of conversation.

I also interviewed my former math analysis teacher from my school, Venia Dardyk. Unfortunately, his teaching partner, my other math analysis teacher, Andrei Pechkovskii recently died of lung cancer. I interviewed nine of my former classmates: Ira (Gertseva) Kazakova, Aleksei Riabinin, Oleg Kazakov, Alexandra Shlyapentokh, Yuliy (Yulik) Baryshnikov, Sergey Popov, Leonid (Lionia) Rozenbaum, Aleksei (Liosha) Saverchenko, and Evgenii (Zhenya) Emelin regarding their experiences as students of Konstantinov math circles and Konstantinov math class in School No. 91. Two of my classmates—Aleksei Riabinin and Oleg Kazakov—were math analysis teachers for the 1979 – 1982 cohort in Math School No. 91. I also interviewed Alexander Shen who was a math analysis teacher for the 1980 – 1983 cohort in Math School No. 91 and then in Math School No. 57.

I decided to organize my text by dividing it into four parts. First, I include a detailed description of Konstantinov's innovative math practices. Based on my first interview, I described my own experiences in Konstantinov's math circle and Math School No. 91. I intertwined my narrative with diverse interviews of Konstantinov (by other people and by me) and my interviews with my former teacher and classmates. The second involves my reflection of Konstantinov's educational philosophy mostly based on diverse interviews with him. The third part involves my discussions of the objections to and challenges of Konstantinov's innovative authorial math pedagogy that I abstracted from various interviews. Finally, the fourth part is dedicated to the history of Konstantinov's pedagogical practice.

I faced the following six major challenges in this project. First, from the vast material that I had, I was forced to select certain materials and stories and leave out others due to a lack space. Second, I needed to provide enough historical and cultural contexts for an international, non-Russian audience. Third, his and my memories were fading; I tried to check them from as many different sources as possible. Fourth, to avoid romanticization of my memories of my math circle and my math school, I tried to consider "dark sides," challenges, and objections of the math education practices open-mindedly. Fifth, I am aware that I, herein, present only a thin slice of the practices that have lasted for more than 50 years, which have been diverse by their nature with numerous participants. I tried to include disagreements and contradictions that I observed and address them in an honest way. Finally, sixth, I am also aware that I am both guided and biased by my particular paradigmatic views in pedagogy that I would characterize as dialogic, multicultural, agentive, and sociocultural (Matusov, 2009; Matusov et al., 2016). I suspect that Konstantinov's authorial math pedagogy may look very different from another paradigmatic vista and I welcome the other viewpoints.

Nikolai N. Konstantinov: Brief Biography

Nikolai Nikolaevich Konstantinov (Figure 1) was born on January 2, 1932, in Moscow. His father was a hereditary honored citizen of Russia and his mother a Georgian noblewoman.

He graduated from the Physics Department of the Moscow State University in 1954 and later received a Ph.D. in physics.

In the 1950s, he started a math circle at Moscow State University (MSU) and since the 1960s in a number of Moscow high schools. He continued working with schools developing special classes with mathematics concentrations and individual approaches to learning. His students went on to win mathematics competitions on all levels and dozens of them became well-known mathematicians.

In 1978, Konstantinov started the Lomonosov tournament, a multi-subject science competition. This tournament has continued every year since then. In 1980, he started the international Tournament of the Towns, which now is organized in over 150 towns in 25 countries.

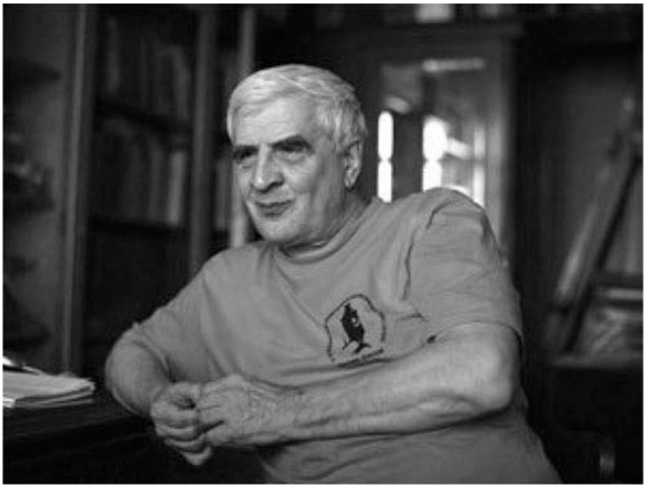


Figure 1. **Nikolai N. Konstantinov**

In 1990, Konstantinov was one of the founders of the Independent University of Moscow, one of the leading institutions of higher learning in mathematics in Russia.

Even now, in his mid-80s, Konstantinov continues working in Moscow High School No. 179 and is an editor of Kvant magazine, a popular Russian science publication.[2]

Part I: Konstantinov's Pedagogical Practice through a Student's Experiences, Observations, and Reflections: 1972 – 1979

Math Facultative in My Local School

Sometime in the 6th grade (1972) in my No. 145 Moscow local public school, my math teacher Galina Antonovna Bondarenko invited us, her students, to join a "math facultative" (i.e., an extracurricular afterschool math club). I liked math and joined the facultative. Galina Antonovna promised us "fun math problems." I was not sure what she meant by that. Before this math facultative, she organized school math Olympics and invited my classmates and me for local district math competitions. The math problems during these math challenges were more difficult than the regular math problems that I had experienced in my math classes. However, they were not fun. Usually, I was able to solve them within 10 – 20 minutes, maximum. I thought I was good in math because I could solve math problems faster than many of my classmates and I often got As on math tests. I guess I liked to be good at math. Most of my classmates hated math.

In my 6th grade math facultative, I faced completely different math. Many problems of our math facultative were fun. Often these problems did not require much knowledge, technical skill, or memory of some theorems that we studied in school. Rather, they required us to find some interesting, creative, and unusual tricks or twists that always were there "on the surface," but would somehow escape my initial attention. In addition, the most interesting and fun math problems were not easy to solve. They required a lot of time, effort, and dealing with the frustration of not being able to solve them. Some problems I could not solve entirely and it surprised me. I knew that I had enough knowledge to solve them—I trusted my teacher, Galina Antonovna, who gave them to us, 6th and 7th graders, but I just could not solve them. I had an ambivalent feeling about this. This sense of prolonged frustration of my failing efforts was new to me. I hated it and I loved it. The hate is easy to explain—most people dislike when their prolonged efforts fail. Additionally, it was rather adaptive for us to give up, to save face, and our emotional well-being, as well as to interrupt useless and hopeless efforts on impossible tasks. It was not as if you would drop it, if you could not work it out in 20 minutes. We faced problems that took a long time to solve, if you were able to solve them at all. As a child at 12, I liked that. It was such a nice moment, when some interesting idea suddenly appeared. Even if this idea did not work, it was interesting and new. And if it was a good idea, that was even better, of course.

Actually, I loved this prolonged frustration. The longer and tougher the prior efforts and frustration were, the higher was my satisfaction when I could find a solution. Additionally, often (but not always) the longer time and more efforts I had to spend on solving the problem, the more aesthetically beautiful was the solution. The perseverance of solving interesting math problems gave me new, mathematical fantasies that I had never experienced before. At that age, I had fantasies of imagining a medieval kingdom that my friends and I ran. My friends and I had science fantasies of inventing a time machine to go to the Tsarist time at the beginning of the 20th century, to escape on a ship to the Chatham Island, south of Australia. We fantasized we were pioneering geographic explorers discovering new lands in our neighborhood. We had military fantasies. We had sexual and romantic fantasies. However, my friends and I had never had math fantasies of solving math problems before. Now I had them. The world became mathematized for me (cf. Lave, 1992). I do not remember specific examples by now,[3] but this is how Konstantinov described a similar mathematization of the world in his childhood:

> … in Turgenev's[4] short story "Bezhin Meadow," I remember only the first sentence: "It was a beautiful July day, one of

> those days that come only after a long spell of settled weather." That sentence got me thinking: if there has been a long spell of settled weather, does that mean that tomorrow will be exactly the same kind of day? If so, it will again be one of those days after a long spell of settled weather, but then the next day will also be the same kind! How many such days will there be? It turns into an infinite string. In other words, already at that time I had sensed a mathematical fact, mathematical induction. Everything else from the story I have forgotten. Although I did reread it recently—there are a lot of interesting things in there. (Leenson, 2012).

I thought about my interesting, fun, but very challenging math problems during my chores: washing dishes, taking the garbage out, shopping for bread, cleaning my room, waiting in line, and so on. I thought about my interesting, fun, but very challenging math problems before falling asleep or walking out alone. I could daydream solving my math problems during boring school lessons or when I had nothing else to do. I could forget about fights or difficult and unpleasant days by escaping into my fun math problems. Finally, trying to solve interesting, fun, tricky math problems for hours, days, weeks, and even months made me feel important and adult.

Sometimes I asked my parents and my brother for help and they contacted our math advanced relatives when they could not help me. I was especially proud to be able to solve tough problems that my parents and math advanced relatives—professional physicists, theoretical (my second cousin) and experimental (my second uncle)—could not solve. I remember at least two of such problems.

Math Problem #1: Extinguishing Fire

> *Problem: There is a village hut on fire. A fire engine has to come from the fire station to the river to get water and to go to the hut on fire to extinguish it. Find the shortest pathway from the fire station to the hut through the river, if the hut and the fire station are located on the same side of the river* (see Figure 2).

I spent a lot of time, many days, thinking about this problem and finally I solved it by using the river as the symmetry line. The

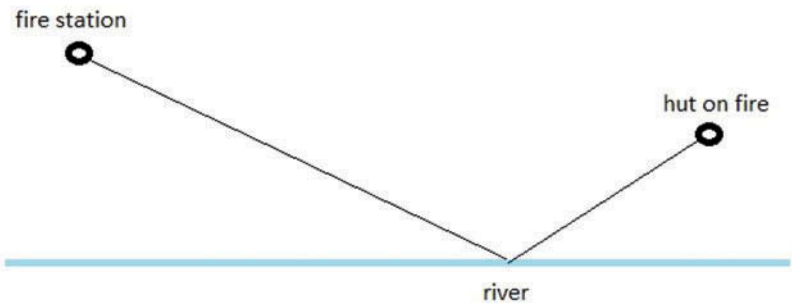


Figure 2. **Find the shortest pathway from the fire station to the hut on fire via the river**

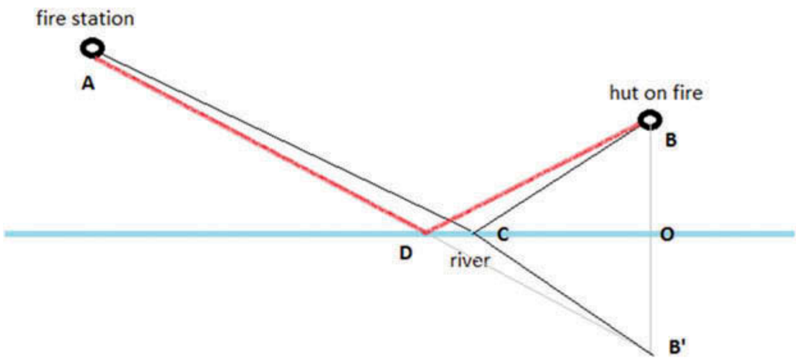


Figure 3. **The shortest pathway (the red line) is defined by the symmetry inversion of the hut on fire (point B') using the river as the symmetry line**

straight line between the fire station (point A) and the inversion image of the hut on fire provides the direction for the shortest pathway (see Figure 3):

The length of any pathway from the fire station (A) to the hut of fire (B) is the same as the length from the fire station (A) to the inverted hut on fire (B') because CB = CB', or AC + CB = AC + CB'. The ADB' length is shortest; it is the straight line and a straight line is always the shortest distance on a flat surface.

I was very proud to show my solution at the math facultative. I remember that explaining this solution took me a while before

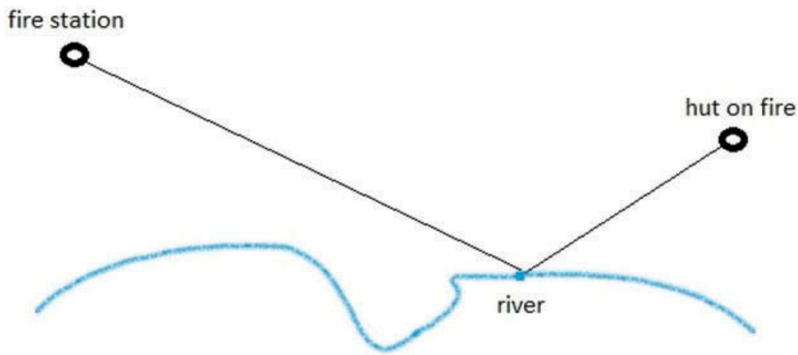


Figure 4. **Find the shortest pathway from the fire station to the hut on fire via the curved river**

some of my classmates understood it. I was very excited about the beauty of this geometric solution and proud that I could come to it myself. The beauty of it was that after making the symmetry inversion, the solution becomes self-evident. However, something bothered me about this solution. It took me several weeks to articulate this new problem. Rivers are never straight lines. They are curved. When the river is curved the linear symmetry inversion solution that I came up with apparently stopped working (see Figure 4):

I spent a lot of time trying to solve this problem but I could not. I asked for help from my parents and relatives but they could not help. I showed this new problem to my math teacher Galina Antonovna at our math facultative. She praised me for inventing this new very interesting problem but she said that she did not know how to solve it. She said that it was a serious problem, probably worthy of professional mathematicians. She also recommended that I try to solve it with a simple curve like a circle—the river is a circle arc. I tried my teacher's suggestion but I still could not solve it. Nevertheless, I was very proud of myself and thankful to my teacher, who recognized the seriousness and importance of my new math problem. In his 2010 interview with Dorichenko, Konstantinov pointed out the common mistake that many math teachers do by trying to make math problems non-serious and overly-playful:

> …math didn't seem like a serious science to me at that time. The math circles had some hares jumping around, red, green and white ones, and you had to prove that there were even number of red ones. All of these problems struck me in some way as child's play. It was strange to spend time on this. Only later did I realize that these, of course, were exercises. But I think that people who misuse such children's narratives do not fully understanding psychology. Kids like it when their activities look grownup (Dorichenko, 2010).

That was definitely true for me. As previously mentioned, as a child in my math facultative, I wanted to feel that I was involved in very serious adult business. Galina Antonovna provided us with serious math problems and not infantilized entertaining problems about "colorful bunnies."

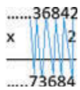
Math Problem #2: Doubling Number

> *Problem: There is an integral number. When the first digit of the number from the left is moved to the right, the new number becomes twice smaller than the original number. Find the original number.*

I called on the phone to my second cousin, who was a theoretical physicist, for help. He tried to address the problem by representing the original and transformed numbers as a sum of their digit multiplied by 10 in the corresponding degree (e.g., $9371 = 9 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 1 \times 10^\circ$). He represented the original number and the transformed number in the following way: $a_n \times 10^n + a_{n-1} \times 10^{n-1} + a_{n-2} \times 10^{n-2} + \ldots + a_1 \times 10^1 + a_0 = 2 \times (a_{n-1} \times 10^n + a_{n-2} \times 10^{n-1} + a_{n-3} \times 10^{n-2} + \ldots + a_0 \times 10^1 + a_n)$. It took me awhile to understand this concept over the phone but with the help of my father, I got it and became very excited. However, in a few more days, I realized that it was a dead-end, although not completely.

My second cousin's approach helped me represent integer numbers of unknown length that I had never done before. I wrote the original number A_1 as: $A_1 = a_n a_{n-1} a_{n-2} \ldots a_1 a_0$ and the transformed number A_2 as: $A_2 = a_{n-1}a_{n-2}\ldots a_1a_0a_n$. Alternatively, I represented the problem as:

$$\begin{array}{lr} & a_{n-1}a_{n-2}\ldots a_1a_0a_n \\ \times & 2 \\ \hline & a_na_{n-1}a_{n-2}\ldots a_1a_0 \end{array}$$



My solution became evident when I started to consider the type of number was a_n. Since A_2 is twice bigger than A_1, it means A_1 is even, and, thus, a_n is even because even numbers always end with an even digit. However, a_n cannot be 0 because integer numbers to do not start with zeros. Therefore, a_n can be either between 1 and 9. By selecting 2 (and then all other numbers) for a_n, I was able to make the reverse engineering of the original number A_1. For example, if $a_n = 2$, then $a_0 = 4$; if $a_0 = 4$, then $a_1 = 8$; and so on. However, there is a complication when the result of doubling becomes more than 9; 1 has to be added to the next digit. For example, $a_2 = 6$, but $a_3 = 3$. Finally, when the digit becomes equal 2 without carrying over to another digit (i.e., not 12), it means this digit is a_n—the digit we started with. Therefore, if $a_n = 2$, then the original number is 210526315789473684, and n = 17. If a_n is 1, then the original number is 105263157894736842, but it should be rejected because after moving the first digit back, the new number starts with 0, 052631578947368421. If a_n is 3, then the original number is 315789473684210526, and n = 17. If a_n is 4, then the original number is 421052631578947368, and n = 17. If a_n is 5, then the original number is 526315789473684210, and n = 17. If a_n is 6, then the original number becomes infinite (636842) 1578947368421052, and, thus, should be rejected as a possible solution. If a_n is 7, then the original number is 736842105263157894, and n = 17. If a_n is 8, then the original number is 842105263157894736, and n = 17. Finally, when a_n is 9, then the original number is 473684210526315789, and n = 17. There are only seven possible answers (of course, each number can be "doubled," "tripled"—e.g., 210526315789473684210526315789473684—and so on but I was not interested in it). Additionally, noticed that the numbers create a rotating pattern of the same digits in the same order, except for $a_n = 6$.

Again, I presented this elegant solution at our math facultative. It generated many interesting discussions, including new math problems of why the numbers rotated and why $a_n = 6$ was an exception. Unfortunately, I do not remember if we solved these new problems or not.

In addition, unfortunately, I do not remember other interesting and fun problems or beautiful solutions developed by my other classmates for problems that I did not solve, but I am sure that was the case. I attended Galina Antonovna's math facultative for two years. Regrettably, I do not know if Galina Antonovna knew Konstantinov and his pedagogical practices but her math facultative pedagogy was suspiciously similar to Konstantinov's pedagogy.

I often think about how a series of lucky meetings with the right people at the right time at the right place shaped my education. This luck shaped not only my education but my inclinations, interests, passions, and probably phobias (in the latter case, it was probably my bad luck of encountering wrong people). Konstantinov described this phenomenon in the following way:

> …if one analyzes what a person is drawn to, one can understand a great deal. After all, people are different.… If a person has many kinds of skills, a choice may be made at random. For example, a schoolchild places first in a chess tournament. This makes a powerful impression on him.[5] He starts going to a chess circle, develops these skills in himself, while the other skills are not developed. The first impression is very important here: it can change the direction of development. It is important what a person first has a proclivity for (Leenson, 2012).

Konstantinov's Math Circles

I think that Konstantinov's math circles were the heart of his innovative pedagogy because it was completely voluntary, which is both the blessing and the curse, as I will discuss later in my analysis.

At the end of 6th grade, Galina Antonovna encouraged me to attend Moscow city math Olympics at the Moscow State University (MSU). At the MSU math competition, the math problems were interesting but much less fun for me for several reasons. One reason was that the time for solving the problems was very limited. I did not like the time pressure. It made the process of solving less fun for me. It felt to me more like school. Second, math competition reduced a sense of a community and collaboration. We were competing against each other. Solving a math problem meant putting down my math colleagues who did not solve it. In contrast, in my math facultative, we were learning from each other and enjoyed the beauty of each other's solutions. Of course, I liked to win and I liked to be first, but at the same time it made me feel pity for all those who were "below" or "behind" me. But deep down, I felt that it was both unjust and untrue. My performance on the city math Olympics was not as spectacular as at my school or my district levels, but by that time, I had learned that it was not very important for me. Most importantly, for me, I met some interesting kids who also were also interested in math. I met my community. But even more, I picked up an invitation to Konstantinov's math circles that had a very appealing and unusual announcement, something such as this:

> "An invitation is extended to 8th-graders, as well as the bravest 7th-graders." In that kind of free style. "The circle will be TREMENDOUSLY interesting." School kids liked the announcement very much and they began to refine it. For example, it said "the bravest 7th-graders," and they added: "but don't forget to insure your life." The announcement was enriched with the ideas of various people, and as a result everyone read it with interest (Konstantinov et al., 2002, p. 44).

I signed up for them at the city Olympics. My parents were ambivalent for my engagement in the math circles. They liked my dedication but were concerned by my traveling over an hour to the math circle to the other side of Moscow city by myself in the afternoon; I was coming home in the late evening (it was very dark, especially in winter). Interestingly enough, in my interview with Nikolai N. Konstantinov, he remembers that his parents were also worried for him, as a young teenager, traveling alone to his MSU circle. His parents felt better when a neighboring girl also traveled to the same circle and she and their son could travel together. It is ironic as a teen boy traveling with a young teen girl could make the travel more dangerous rather than safer.

As in my local math facultative, Konstantinov's math circles involved many interesting and fun math problems that we were trying to solve there and at home. The math circle I attended was multiage as I saw younger and older kids there. We met at the zoology museum, passing by exhibits with bone remains of many interesting animals, including a skeleton of a mammoth. We were sitting in big rows, organized as arcs of an amphitheater centered on the long blackboard. This was an old university auditorium. It felt very adult to me. We solved diverse problems.

Initially, I could not remember these math problems in the circle but after interviewing my No. 91 school classmate Alexei Riabinin, who also attended Konstantinov's math circle around those years (but not with me), my memory started coming back. There were many different math problems about a goat tied to one or many poles—we needed to figure out the area of grass that the goat would eat. I remember struggling with a modified problem I invented for myself of figuring out how to make the goat eat the area of a triangle. Another problem I remember was about nuts: all nuts are weighted the same except one, which was empty and thus lighter—how many minimum weighting times can be done to find out the empty nut by using a simple scale. The total number of the nuts varied (e.g., 7, 9, 13) but the most interesting task for us was to solve the problem in general, with N as the number of nuts. Our teachers were college students who looked VERY old and adult-like to me. In the class, there were about 30 – 40 kids and 5 – 7 college students, our teachers; most, if not all, were males. We raised our hand to call for a teacher when we thought we solved the problem. A college student came to us "to accept"—testing the solution. The process of "accepting" our solution involved our teachers carefully listening to us, praising the most "beautiful" and promising places in solution and finding possible holes in it. If our solution could not survive their challenges and failed, the teacher encouraged us to keep solving it. Sometimes we tried to solve problems together but I do not remember any lasting friendships from my math circle.

Sometimes, our teachers lectured us about some interesting mathematical issues. I cannot remember the topics but again my classmate Alexei Riabinin reported one topic that he could remember. It was about a proof by a famous German mathematician Georg Cantor that points on a part of a lane cannot be counted (i.e., the reals are uncountable[6]).

I think I saw Nikolai Nikolaevich Konstantinov back then; he was an athletic middle-aged nice looking man in jeans, who was older than my teachers in the math circle were. He talked with kids and listened to them very carefully and with a lot of interest. I immediately noticed that while interacting with kids, he did not have a patronizing attitude common to many adults. He was genuinely interested in kids and math. Also, he was apparently rather respectful to the college students who also showed their respect to him. I remember that sometimes he interrupted their lecture with a question or an interesting comment.

The math circle's session lasted about 1.5 – 2 hours. We got problems to solve at home but we did not expect to solve them all. It did not feel like school homework as it did not feel tedious and hovering. We met once a week but could always skip a meeting. Math circles were voluntary.

It was a very new and exciting concept for me—voluntary education! I studied because I liked it and not because I was forced or pressured to study. I felt as though I lived in another country, different from the country where many of my classmates were living. I sensed freedom of self-realization. It was an oasis of freedom and inspiration in the otherwise boring, prescribed, and controlled life of a young Soviet teenager in Moscow, in the post-totalitarian stagnating USSR. Here is how my No. 91 school classmate, Yulik Baryshnikov, describes his feelings of freedom:

> . . .the math circle (in the human-sciences building of MSU) was a bright spot in a pretty dull life. People behaved freely, the texts read like samizdat[7] ('Witch Ania is flying a broom at 100 km per hour. . .'). And the very idea that one could describe and determine everything and could arrive at an unpredictable conclusion seemed liberating. I think that was the genius of NN [Konstantinov]—those who were not brought in by their parents, or by some passion for science from earliest childhood, were pulled in precisely by this. That is what happened with me, anyway."

Konstantinov's Math School No. 91

In 7th grade, at one of my MSU math circle's sessions, there was an announcement regarding an open admission to Moscow Math Schools No. 91 and 57. I remember asking one of my math teachers at the circle, what was a math school and he replied, "It is school where math classes are taught like in our math circle." Wow! I liked that! I wanted it badly! My parents were much less enthusiastic, as they did not want me to travel to Moscow Schools No. 91 or 57 as the schools were about 45 – 55 minutes away from my home. As they reported to me later, they secretly hoped that I would not pass the entrance tests. Some parents pushed their children to the math schools but some parents highly resisted them as reported by Konstantinov to me in an interview.

Konstantinov's Entrance Tests for Math Schools

We had exhausting entrance math tests: 5 – 6 weekly rounds lasted for about month and a half. Each round involved a meeting in a math school (I think it was Moscow school No. 57) and we worked on 5 – 6 math problems for two or three hours. We also got 5 – 6 problems to solve at home. During the test, we had to solve problems and defend our solutions of these and home problems to the teachers and college students. Usually, one or two problems were VERY easy and then the difficulty level mounted. The last one or two problems were usually super difficult. Konstantinov confirmed that in his interview with me, "Of course that was the case. It is very important that a person have something to work out at first, otherwise he just falls into a stupor." At home we could consult with whoever we could—I showed my home problem at my math facultative and to my parents and relatives, but it rarely helped. By the end of the rounds—it was, I think, in late April 1974—I was sure that I would not be accepted. I remember that I solved about 60% – 70% of the problems, fully or partially. I felt I was far from being successful.

My parents tried to prepare me for the worse (for me, not for them). They found a local school not far from where we lived where a math teacher offered enhanced math instruction. However, I did not want to have enhanced math instruction. In my vague sense, I wanted math freedom, math self-realization, math community, and college students teaching me. I wanted voluntary education. I wanted freedom of curricular choice. I could not explain all of that to my parents back then but I felt it deeply. As the moment of truth was coming, I was getting more and more unhappy and desperate waiting for a decision letter.

To my big surprise and relief, I got accepted! I suspected that everyone who did not drop by the last fifth or sixth round of selection was accepted and later Konstantinov confirmed my suspicion in an interview with me:

EM: It seemed to me that they selected people not based on the number of solved problems but based on the persistence with which a person simply continued to work. Is that true or not? If that's true, then it's very unusual, because usually the selection is based on the amount of points scored, whereas here it depended on interest and perseverance.

NK: I can say why that was the case.[8] Because everyone was pretty bad at solving problems. If the problems were easy, then it could have been based on the amount, but that was not the case (Conversation between EM and NK, part 1, October 30, 2016).

I had a choice of going either to School No. 57 or 91 for my 8th, 9th, and 10th grades.[9] I did not care about either of these schools. I was happy to go to any math school of Konstantinov's. My parents examined these two choices and selected Moscow School No. 91 because it was within walking distance from where my grandma and her sister lived. My parents decided that I should live there and visit them on weekends. Initially, I did not like this arrangement because it meant that I could not see my local friends during the week, but I surrendered because it was my contribution to the compromise with their concerns. As time passed by, I loved the arrangement.

Description of Math Classes in the No. 91 Moscow Math School

In our math class, there were about 30 students, mostly boys (see Figure 5). The class involved diverse ethnicities: Russian, Jewish,

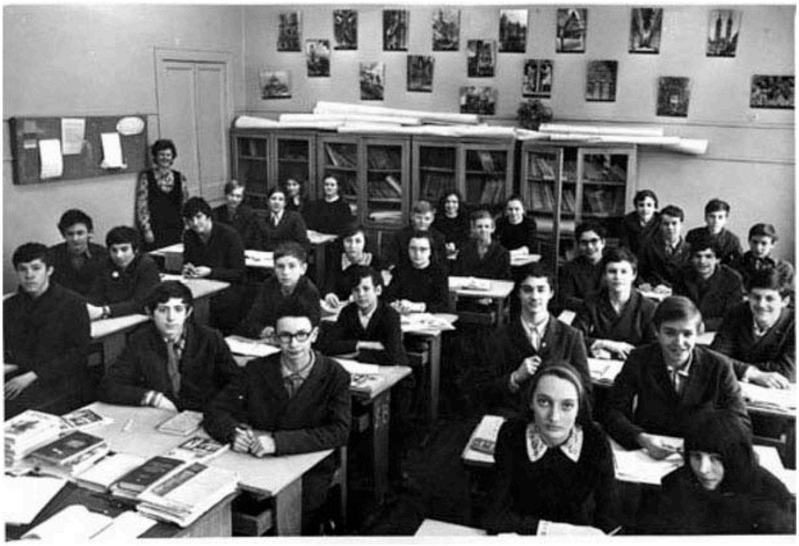


Figure 5. **8th grade of our math class with our home history teacher Zara Moiseevna Lubavina, 1975, the history classroom**

Belorussian, and Tatar. Some of the students were children of working class families and some of intelligentsia.[10]

We had two math classes with very different organizations, each having four 45-min class periods per week. The first math class—general math –was taught by Nina Iurievna Vaisman. I am not sure, but back then I (and some of my classmates) thought she worked at the Moscow State University. Later, in preparation of this text, I found that she used to work in Moscow Math School No. 2 as a math teacher and vice-principal 1966 – 1973 (Smirnov, 2004) and then 1973 – 2003 at the All-Russia Multidisciplinary Correspondence School.[11] She came specifically for our class and did not teach other classes in our school. I asked Konstantinov and Venia Dardyk about her, my other math teacher about whom I am going to discuss in the following section. Neither of them knew Nina Iurievna Vaisman. My guess is that Vladimir Mironovich Sapoznikov, another math teacher in School No. 91, brought Nina Iurievna to us. It seemed that for some reasons, Vladimir Mironovich could not teach our class and invited Nina Iurievna Vaisman to replace him. Interestingly enough, Vladimir Mironovich Sapozhnikov taught all other math classes in School No. 91 before and after our class until his death in 2004. He also taught math in all non-math classes in our school. As far as I remember, Nina Iurievna taught us algebra, linear algebra, trigonometry, and stereometry (i.e., solid geometry). Her instruction was rather conventional: mainly lecturing combined with assignments. However, her curriculum was very advanced in comparison with a conventional math curriculum. I do not remember being very excited about her lessons, but my classmate Yulik Baryshnikov commented highly about her instruction, "Nina Iur'evna (who conducted our programmed math) knew how, in my opinion, to structure her teaching hours, much more dramatically [than the math analysis class] despite the total dullness of her subject."

The other math class, "math analysis" (calculus), involved several diverse math disciplines: the theory of numbers, geometry, calculus, and programming (i.e., computer science). The class was organized non-traditionally. The class, which was two class periods together, was taught by college students. We had two permanent college students Andrei Pechkovskii and Venia Dardyk. As our permanent teachers, they were paid about 20 rubles (about $30) a month by the school and the school administration accommodated their availability constrains in the course schedule. We also had 5 – 7 other college students, friends of Andrei and Venia (in contrast to the other teachers, we called them by their first names), who came to us for our math analysis classes irregularly. I remember only a few of them by names: Sasha Romanov (he was older), Roma Deminshtein, and Leva Fridlender. I do not remember any female college students coming to help Venia and Andrei. We had between 2 and 5 college students as our teachers in our math analysis class. At that time, both Venia and Andrei attended the Moscow Automobile-Road Institute (College, MADI) that I attended after graduation from our school. Venia was alumni of Math School No. 7 and Andrei was alumni of our Math School No. 91. From time-to-time, Nikolai Nikolaevich Konstantinov visited our math analysis class to talk with our teachers and us and observe how things were going. Like any other Soviet school, we had the cohort system staying with our classmates all the time.

I asked Konstantinov how he selected math analysis teachers for math circles and, especially, math schools. He replied that initially they were his colleagues from the Moscow State University but then they became alumni of math circles and math schools. The main criterion was not so much their knowledge of math but interaction with the children,

> Of course, the main thing for me was the interaction with pupils. And I watched how my candidates knew how to interact, they knew how to talk. That was the criterion: I saw that this person would be interesting for the pupils, and the pupils would be interesting for him and, therefore, I could try to invite him. That was the way Pechkovskii and Dardyk were—they were a perfect fit in that respect. It was clear that they knew how to interact, that it would be interesting for the kids to talk with them, and they would find it interesting to talk with the kids. And they had enough math qualifications for the initial period. They would not confuse a direct theorem with the reverse one; in short, they would understand what they were doing (Conversation between EM and NK Matusov, October 30, 2016).

Konstantinov elaborated:

> Relationships between [regular] teachers and pupils are like relationships between parents and children. Relationships between undergraduates and pupils are like relationships between older and younger siblings. But they are different things. As a result, undergraduates who are the heads of math circles, organizers of Olympiads and math-class instructors have an opportunity for such mutual understanding with their pupils that is not available to older teachers. (Imaikin, 1998)

Our math analysis class was run as Konstantinov's math circle. We got famous leaflets ("listochki"—"little sheets") with brief definitions of the key math concepts and numerous math problems defining a broad curriculum unit we studied (see Appendix for an example of a leaflet). We were supposed to solve math problems in class and at home. We could pick any math problem on the leaflet list to solve. We could solve it solo or with our peers. There was not much boundary between class and home. I am not sure about my classmates, but I perceived the math problems as exciting invitations and suggestions rather than tiresome assignments, hovering over me at home. We were not required to solve all the problems. When any of us thought that he, she, or they solved a problem, we called one of our teachers to accept or reject our solution. At times, some of my classmates, including me (see Figure 6), solved very few math problems or were not engaged for a while, preferring to pursue our non-math interests.

In testing our solutions, our math analysis teachers focused on at least three issues: a) holes in our solutions, b) correctness of our solution points, and c) beauty of our solution (i.e., mathematic aesthetics and creativity). When the solution was accepted by our math teacher, he would mark it on the special spreadsheet. Sometimes our teachers were making helpful suggestions but

Figure 6. **Andrei Pechkovskii (on the left) considers our solution of a math problem (Igor Tsarkov, in the middle, and me, on the right). In front of us Alexandra Shlyapentokh working on her math problem. 1975, 9th grade, math analysis class**

often they did not interrupt or impose their own approaches or solutions on us. When many of us got stuck on some difficulties, they offered us mini-lectures. During these mini-lectures,

> every issue is discussed for as long as it requires. There is no rush. It does not happen that we have to finish a topic by a certain date and therefore we would have to move faster, while some people would get a 'two' [failing grade], some a 'three' [fair], as we would hurry on. There was none of that. Every item was considered thoroughly, for as much time as necessary. And that is precisely the style that is needed in science (Gubailovskii and Kostinskii, 2004).

Sometimes our math analysis teachers invited some of us to share their solutions with the entire class. Sometimes they offered mini-lectures regarding certain topics. I remember their lecture on math induction. However, we were not required to attend to these lectures and we could mind our own business during their lectures.

As I remember, the class was often, if not always, noisy. As I interviewed Venia Dardyk, he did not remember any problem with the school administration because of that; apparently, the school administration expected that these math analysis classes would be noisy. Sometimes we did not work on math problems but played different games during our math analysis classes, such as the unlimited five-in-a-row tic-tac-toe, racetrack game on a celled sheet, [12] 5-kopec soccer, self-made Monopoly, and card games such as Preferans, Bridge, Blackjack, and so on. Some of these games provoked new interesting math problems in us (e.g., finding winning strategies in the unlimited five-in-a-row tic-tac-toe or in the racetrack game). In my view, this informal play culture was very important for creating and supporting a math culture and a general creative intellectual and educational culture in our class. However, as my classmate Lionia Rozenbaum remembers in his interview, he spent most of his time solving math problems in math analysis classes because, as he put it, it was his main reason to come to the math school. This was my memory as well. I think the serious culture of solving interesting math problems and playful culture of games created an important synergy.

Lionia Rozenbaum remembers that our math analysis teachers had a bumpy beginning.[13] He remembers that at the beginning, Venia demanded an order in the class that we were not willing to provide. At some point, Venia declared ending their pedagogical liberalism and establishing the Junta with its strict order and oppressions. It was around the year anniversary of the 1972 coup in Chili by the military Junta. In response, a small group of my classmates started publishing a self-made journal "The Voice of the Junta," mocking the new order (a few issues were collectively published). Soon, Venia and Andrei relaxed. There were no repressions; they did not contact the school administration or our parents and did not try to hit us with bad grades. It seems to me that Venia and Andrei started appreciating the creative mess and noise in our class and learned how to manage it by negotiating and reasoning with us. I think that they changed their pedagogical expectations to focus on healthy class ecology rather than on the perfect and efficient order.

Our math analysis teachers gradually became very tolerant to noise, to our engaging in other math and non-math activities during the class, and to not listening to their mini-lectures or other students' presentations. The only demand from our math teachers was not to disturb other classmates working on their math problems, sharing/testing their solutions with the math teachers, or listening to the math teachers' or peers' mini-lectures. When these disturbances happened, Venia and Andrei firmly and respectfully asked the disturbing students to stop, explaining the negative effect on other people in the class, and as far as I remember those students often, if not always, apologized and stopped or reduced their disturbing activities to Venia and Andrei's satisfaction. Out math teachers had a lot of respect in our class, we did not want to upset them and we seemed to understand their concerns as legitimate. Additionally, math analysis was often the last class in the day, which gave those of us who did not want to engage in math or comply with our teachers' demands, freedom to leave the school. Finally, I remember that at times some of us, the students, promoted order in the class by either restraining ourselves or by restraining others who were disturbing the class. I do not remember having any big conflict with our math analysis teachers ever about the class order (or about any other issue).

Not all of the students were interested in solving problems all the time, but I do not remember that they ever forced us to do that. As far as I remember, our math analysis teachers sabotaged our final grades; most of us got an A regardless of how many math problems we solved. Sometimes, when our math analysis classes were the last classes, some of us, including me, left the class to go outside or even go home.[14]

No one ever punished us for that. The atmosphere in our math analysis classes was one of voluntarism. I always knew that when I was solving math problems, it was mostly because I wanted to do that and because I was interested in math problems. The only external pressure I felt was about not upsetting my math teachers, Venia and Andrei, and sometimes I felt I did not want to fall too much behind my classmates in my math problem solving. On average, I spent numerous hours per week thinking about math problems of my choice. Many of my former classmates claimed that our academic motivation was not based on rewards and punishments but mostly on our genuine interest in math and other subjects, on informal rich intellectual culture, and, probably, a bit on vanity (at lease for some of us at some time).

In Figure 5, on the first desk in the middle row, one can see a book between the two boys. This was not a history textbook but a math book by a famous American math popularizer Martin Gardner.[15] I cannot remember who introduced this and other math and science books to us, but these books circulated in our class. I remember reading many books by Martin Gardner; Lewis Carroll's book *A Tangled Tale*; Yakov I. Perelman's book *Mathematics Can Be Fun*; Richard Feynman's book *The Feynman Lectures on Physics*; and so on. Later political and literary samizdat forbidden books joined our class circulation.

My classmates created a very important environment for me, which was both supportive and competitive. As I mentioned previously, I did not like competitions because even when I won, I felt sorry for those who lost. I liked it more when we were collegially supporting each other's math efforts rather than being competitive. We engaged in many rich intellectual and ontological discussions about math, physics, literature, art, poetry, music, politics, and other spheres of life. For example, in the 8th grade, my then friend Yulik Baryshnikov and I created a new science of studying how and why people got tired—we called it "ustologia" (literally "tirelogy"). In one of our physics lessons, we studied the mechanical definition of work as force multiplied by distance. For us, this definition contradicted our everyday experience: a person who holds a heavy bag obviously works but according to this definition, the mechanical work is zero, because the heavy bag did not move. Our then physics teacher sent us to our biology teacher because he told us that biological work is different from the mechanical concept of work: human muscles work under stress even when nothing moves. The fact that the physics' definition of work did not apply to biology did not make sense to us and we decided to investigate that by creating the new science of ustologia.

Another contradiction that we noticed in physics was the mechanical definition of motion. Motion is always relative in the classical Newtonian mechanics: it is relative to an observer. However, people say that the Earth rotates around the Sun and not the other way around. It became apparent for us that this statement contradicted the Newtonian mechanics. We reckoned that it may be easier to describe patterns of movements of the planets by observing from the Sun rather than from the Earth, but it is not true that the Earth rotates around the Sun and not the other way around (this issue is discussed in Konstantinov, 2007, p. 17). Other subjects and the entire life became open for our peer discussions. What I LOVED back then is that our (or at least mine) world perception changed. Before the math school, I felt that my role as a good youngster was to appropriate deeply what generations before me developed as knowledge; my role was to join *their* Knowledge and *their* Truth. My own opinions and thinking had been irrelevant and unimportant. In my math school, together with my peers and some teachers, I learned that my role as a good youngster was to try to critically make sense in open discussions with my peers and teachers of everything that I saw and learned. Everything had to be tested with my peers (and beyond) and was forever testable (cf. Morson, 2004). Any ready-made "solution" produced by the authoritative culture of the past has to be tested in the same way as our math analysis teachers tested our math solutions.

In the following section, I want to provide a "thick" personal description (cf. Geertz, 1973) of our math pedagogical practices in the school, which were the most memorable and arguably the most influential for me (and probably other students).

In his 2010 interview to Sergey Dorichenko for the *Kvant* journal, Konstantinov insisted that intellectual ideas must be "deeply felt" rather than just intellectually understood. I was faced with this issue in my math analysis class. The most memorable event in my math analysis classes involved my math analysis teacher Venia Dardyk and the limit-based definition of the continuous function during our calculus unit. The teacher gave us a leaflet with a definition of continuous function and a list of interesting math problems (easy and very difficult). I could not move beyond the definition. The following is an approximate replication:

> The function f is said to be continuous at the point c if the following holds: For any number $\varepsilon > 0$, however small, there exists some number $\delta > 0$ such that for all x in the domain of f with $c - \delta < x < c + \delta$, the value of $f(x)$ satisfies f(c)—ε < f(x) < f(c) + ε. Alternatively written: Given subsets I, D of R, continuity of f: I → D at c □ I means that for every $\varepsilon > 0$ there exists a $\delta > 0$ such that for all x □ I.[16]

I remember understanding the mathematical sense of the statements but I could not understand its derivation. I read the definition numerous times but it did not make much sense. Why did mathematicians develop such a cumbersome contrived definition?! One could have developed a much easier one, for example, "Function f is continuous if one can follow it with a pencil without any jump!" I explained my difficulty to my math analysis teacher Venia. I remember, Venia was listening to me very attentively and replied, "Zhenya [my nickname in Russian], I don't understand your struggle but I think it's very important. I suggest you spend time thinking about that. Do not rush to try to solve the problems on the continuous function on the leaflet. Go and think more about your struggle" (this is my recollection and not an exact quote). I spent a few days thinking about it, until suddenly I came to a conclusion that this limit-based definition was developed for blind people who could not perceive a non-jumping pencil following the line of the function. Applying it for the sightless, the limit-based definition made perfect sense to me. After this realization, I did not need to read the text of the definition to reproduce it by thinking about blind people. I was excited. I rushed to share this discovery with Venia. Again, Venia listened to me very attentively and said, "Good. Again, I'm not sure I understand what you are saying but I have no doubts that you discovered something very important." I started solving math problems on the continuous and discontinuous functions on the leaflets. I really appreciated Venia's serious and supporting reply to my struggle. My teacher's reply produced a community behind me that, in its own turn, promoted my math voice and my authorial agency beyond math.

Konstantinov emphasized that math has to be deeply felt and conceptually understood, which often is reduced to procedural manipulation of mathematical symbols in conventional math lessons,

NK: . . . my mother's . . . father, my grandfather, was a math teacher. He had a big workload, a ton, and to top it off his little girl was bothering him with some silly questions. She became accustomed in the early grades to do these problems: 5 + 7 = 12. Everything is clear. One can understand what is being asked, and what the answer is. But when algebra starts, A + B appears. So what is A + B? There is no answer. It seems like something that makes no sense. What is the meaning of this? And my grandmother, her mother, told him: "Well, explain it to her, why doesn't the poor girl understand?" He tries to explain, but he has no time, and she doesn't understand anyway. What kind of strange problem is this, A + B? What to do? But later she understood everything and began to love math. But the first misunderstanding was obvious. . . . Many pupils can't understand [algebraic

> formulas]. . .. the only thing they could think of was to learn the exam question paper by heart (Conversation between EM and NK, part 2, November 2, 2016).

In our math analysis classes, we were spending time on "re-discovering the wheel" by trying to solve math problems by ourselves that have been historically solved. As my classmate Alexandra Shlyapentokh wrote to me, "You are asking for ancient memories. The only thing I remember distinctly is that everything took a long time. I think we were reproving by ourselves all the standard theorems of one variable analysis. One problem does stand out: showing the equivalence of the epsilon-delta definition of continuity and the definition via sequences. I was very proud of myself when I figured out the proof. ☺" Our teachers respected us taking time on our deep authorial learning.

My mind always has been very grounded, earthly, and embodied. I could not understand things until I knew their origin, which is a human earthly need for them that gave or could give them birth. "Why bother and who cares" were and are my frequent questions when I am faced with and face problems, issues, and just new information/knowledge. The famous American philosopher of education John Dewey called it "psychologizing" the ready-made knowledge (Dewey, 1956). It can also be called "ontologizing the ready-made knowledge and problems." Let me give another example of this process that came from my classmate Yulik Baryshnikov.

In the 10th grade, many of my No. 91 school classmates and I participated in math Olympics for high school students organized by several Moscow colleges (called "institutes") including the Moscow State University. Among others, we went to a math Olympics organized by the Moscow Institute (i.e., college) of Steel and Alloys (MISAA, or MISIS in Russian). One math problem was very difficult for me: "In a given arbitrary prism, a vector is made from each side of the prism. Each vector is perpendicular to the prism side and its length is equal to area of the corresponding side. Prove that the sum of all the vectors is zero." I spent a lot of time trying to project the vectors onto three orthogonal flats but failed because my formulae were very confusing to me. Recently, I learned from a math professor, Yuliy Baryshnikov, that it was a version of Stokes' theorem.[17] However, my classmate Yulik Baryshnikov solved this problem easily and beautifully by recognizing that it is a consequence of Newton's first law of motion that can be formulated such that, "If a body is motionless, the sum of all forces applied to it is equal to zero."[18] Indeed, imagine an empty motionless prism with solid sides outside of gravity. There is a gas inside of the prism. The gas pressure is constant everywhere in the prism (otherwise the gas would start moving in the prism but it is also motionless). The gas produces forces on the prism sides. The force is equal to the pressure multiplied by the area of the prism side and is perpendicular to the corresponding side. Since the prism is motionless, it means that the sum of the vector forces is equal to zero, according to Newton's first law of motion. That was Yulik's solution of the Stokes theorem on the MISIS math Olympics. I do not remember if his physics solution was accepted by the MISIS organizers of the Olympics or not, but I loved it. I loved it because: a) Yulik grounded the contrived math problem (and Stokes' theorem) in physical reality; b) he applied physics to solve a math problem; and c) the solution was aesthetically beautiful, reminding me of a lightning bolt moment of suddenly seeing a new reality, a new gestalt. Paradoxically, currently, a Professor of Electrical and Computer Engineering at the University of Illinois, Yuliy Baryshnikov, dislikes his physics solution of the math problem at the MISIS math Olympics. He wrote to me, "Well, I probably read this 'proof' somewhere. And it was a waste: somehow it's absurd to rely on an experimental fact (not to mention fluctuations) where you need reasoning. But it's also wrong to ask kids to prove Stokes' theorem—there is a good chance that some of them have already read it in the textbook." In my view, Professor Yuliy Baryshnikov is unfairly too tough on himself; although, of course, there is an important creative tension between the approaches to proofs in mathematics and physics.

My other memorable unit was regarding computer programming (without computers). At the time, in our 9th grade, Venia was interested in so-called normative algorithms and he introduced us to Markov's replacements/substitutions, which is a special programming language developed by Russian mathematician Andrei Markov, Jr. (it is somewhat similar to Alan Turing's machine, https://en.wikipedia.org/wiki/Markov_algorithm). The Markov algorithms involved transformation of words from some alphabet of symbols using substitution formulas such as a -> b as this will replace all letters "a" in a given word to letters "b". For example, the word "appla" will become "bpplb". There are special symbols that are not a part of any words. Some of these special symbols were used to control the algorithm (e.g., the symbol dot to indicate the end of the algorithm) and some were just special symbols that can be used for programming (e.g., Greek letters). Reading about the original Markov algorithms, I realized that Venia might modify some of the rules. Venia created leaflets for us regarding the Markov algorithms that involved a brief description of the rules and definitions of the Markov algorithms. Venia also created many interesting programming problems such as cutting the entire given word except the first letter, cutting the last letter of the given word, doubling the second letter "a" in the word, making the mirror inversion of the given word, and so on. These were very challenging and fun programming problems. I remember that sometimes we played being a computer to execute a complex algorithm of our peers to ensure that it could work with any given word. We also received leaflets, developed by Venia and Andrei, for studying programming language ALGOL and we had a summer practicum at a Moscow computer center, which allowed us to practice programming with a huge computer NAURI using punch tapes. I remember one problem was to identify the gender of Russian nouns. The whole programming unit was developed by Venia and Andrei independently of Konstantinov. The expectation was that our math analysis teachers/college students would develop their units and corresponding leaflets with definitions and a list of math problems in addition to units and leaflets provided by

Konstantinov. In my judgment, I learned computer programming in Math School No. 91 from Venia and Andrei. They created the solid foundation of my independent learning of numerous programming languages that I have since learned such as FORTRAN, PL, PHP, Moodle, JavaScript, FrontPage, HTML, MySQL, and many others.

Finally, I remember being involved in developing new math problems by constantly modifying and problematizing the math (and non-math, as previously described) problems I had faced in my school. For example, when Venia and Andrei gave us leaflets with arithmetic axioms (see Appendix 1) in the 8th grade, some of my classmates and I tried to develop alternative axioms to see what kind of math could emerge from them. I remember that I especially focused on an inquiry of where this alternative math might exist and be used. In my interview/conversations, some of my classmates also remember inventing new gf.math problems but some did not. It is unclear for me if they did but could not remember or were not involved in making new problems. Neither Konstantinov nor math analysis teachers, whom I interviewed, remember their students inventing new math problems, except Alexander Poddiakov.[19] Additionally, they did not encourage their students to do that. Apparently, Konstantinov and his math teachers did not realize the importance of this activity despite the fact that they themselves have been actively involved in this activity of developing new math problems.

School No. 91: Beyond Math

Our education in Moscow School No. 91 was not limited to mathematics, which I think was a part of Konstantinov's pedagogical desire. We had a lot of other afterschool extracurricular activities. We went on Sunday hiking trips with our math analysis teachers in woods around Moscow, we participated with them in KSP (so-called "The Club of Amateur Singing"), [20] we attended movie theaters and some semi-legal music concerts together, we had parties, and so on. My classmates Alexei Saverchenko remembers:

> I participated in forest hiking and KSP meetings in woods. Without doubts, these trips deepened my life by providing a variety of experiences in my otherwise routine life and they educated me. In these trips, we developed communication skills different from ones occurring in school and learned independence and resilience in emerging extreme situations. This was different from our experiences in the math-school. In contrast to math Olympics, where each was on his/her own against all others, in the trips we learned to cooperate, supporting and helping each other, to overcome difficulties and solve problems together, which was impossible to solve by one person" (email communication, February 28, 2017).

In the summers of 1975 and 1976, between 8th and 9th and between 9th and 10th grades, math summer camps in Estonia, organized by Konstantinov and his colleagues, were available for our math teachers and us. These were economically self-sufficient camps where all the participants—children and adults—worked in local collective farms to earn food and lodging. At night, there were math lectures, singing songs around a fire, and conversations about math, philosophy, art, politics, and just life. For some reason, playing chess was forbidden. I did not go there but my classmates Irina Kazakova (then Gertseva), Aleksei Saverchenko, Yevgeny Yemelin, and Alexei Riabinin were fondly talking about their experiences "at the edge of the world" where math lectures were given and adult mathematicians, students, and high school kids interacted and work together.

Our math teachers introduced us to college students' culture and activities. Some of these activities were politically illegal. Thus, I remember that one of our irregular math analysis teacher's, Liova Fridlender, shared with us that he studied Hebrew, Judaism, and Jewish culture, which was officially and legally forbidden in the USSR. The Soviet State Antisemitism was puzzling for me—I felt it was so unfair!

Also, our schools had several non-math facultatives (informal after-school clubs). I attended a literature club with a literature teacher outside of our school who was a friend our physics teachers. The name of the literature teacher who led this facultative was Lev Iosifovich Sobolev.[21] We met in a physics classroom to discuss famous Russian and Soviet writers such as Gogol (short story "The Nose"), Tynyanov (his novel *The Death of the Vazir Mukhtar* and short stories), Bunin's immigrant poems, and Mikhail Bulgakov (*The Master and Margarita*). The choice of the literary masterpieces was semi-legal and extraordinary. The literature facultative were attended by college students specializing in studying literature. The level of the discussions was mind-blowing for me. The focus was not on studying authoritative ready-made truths about a literary masterpiece but on developing our own unique, authorial, informed, and defended aesthetic tastes, opinions, visions, and judgments (cf. Bakhtin's notion of "internally persuasive discourse," Bakhtin, 1991; Matusov and von Duyke, 2010). My classmate Oleg Kazakov remembers how he visited just one meeting that influenced him for his entire life. At the meeting, Lev Iosifovich articulately and passionately read a fragment from Bulgakov's novel *The Master and Margarita*. Then, he asked the students if there were words or phrases that we did not know or that confused us. As we provided our answers/questions (e.g., what does "Roman centurions" mean?), our teacher told us stories, creating a tasty narrative-type cocoon around Bulgakov's original text and engaging us in deep discussions. Although I often was on the periphery of these discussions, as I could not hold my own on this subject, I developed my own orientations and opinions regarding the discussed fascinating topics. I was mesmerized by new imagery that the teacher revealed to us. For example, I remember he read aloud Tynyanov's story about Decembrists—Russian failed conspirators against the tsar and monarchy at the beginning of the 19th century[22]—and specifically about its leader Pavel Pestel.[23] Our teacher re-read several times Tynyanov's description of Pestel: Pestel often signed his name in French and his stroke on the letter *t* was reminiscent of the sharp knife of a guillotine. In our literature facultative, we discussed this powerful image, apparently communicating Tynyanov's own worries regarding the would-be victory of Pestel's Decembrist Revolt and the following terror that was similar to the French Revolution. Some of us suspected that Tynyanov worried not only about the Decembrist Revolt of 1825 or the French Revolution but about the October Communist Revolution of 1917 and the following terror. I was always amazed how our literature teacher could always abstract something very interesting in each contribution by the college students and my classmates.

Another facultative was the Moscow-famous cinema facultative (kinofak) led by a physics teacher Roman Yakovlevich Guzman. It was a unique phenomenon of life in Moscow. Every Thursday during the school year, we watched and discussed the best Soviet and foreign movies and the art of movie making. We saw a number of outstanding films that were unavailable in the Soviet box office. It was unclear how Roman Yakovlevich managed to obtain these movies and circumvent Soviet censorship. At times, kinofak viewers in the House of Culture "Red Tekstilshchiki" attracted literally hundreds of young moviegoers with their parents and friends. We had hot debates regarding art and politics after watching the movies. I remember that in the early 8th grade, I characterized some of my classmates' ideas (e.g., by Kolya Ukhanov's), expressed at the kinofak discussions, to my parents as "Anti-Soviet." By the end of my 8th grade, I stopped using this term as I started developing my own critical authorial political views and stopped parroting my parents' views and official Soviet Communist propaganda. Later, when I became a physics teacher in one of Moscow schools, I also organized a kinofak in my school in association with Guzman's kinofak. During the Perestroika in the summer of 1988, we somehow persuaded the entire cinema theater to run foreign movies from the Moscow International Cinema Festival. High school students ran the theater (mostly students from School No. 91), and we had children's jury and children's meetings with world-renowned movie directors (e.g., Fellini, Forman).

At the beginning of the 9th grade, we were assigned a new physics teacher Aleksei Yurievich Korostelyov. He was a very dialogic teacher in a very particular way. I would say, he was Socratically dialogic. He constantly challenged any statements we made, which was often very discouraging and annoying for some of us. Thus, I remember he was teaching physics by introducing interesting problems/provocations for us, and then we would discuss an approach. His physics classes were interesting and challenging. One of the problems, I still remember, regarded the question of why in locations where there is a lot of sun, people have black skin, while in the locations where there is a lack of sun, people have white skin. From a straight physics point of view, it should actually be the reverse, because a white surface better reflects the sun and a black surface actually absorbs the sun more. From this straight physics point of view, the reverse would be that black people should live in the places with the lack of sun while white people should live in the places with a lot of sun. Europeans should have been black-skinned, and Africans should have been white-skinned. The puzzle was to determine why this is not the case. The physics teacher had an interesting style of discussion as he demolished almost any position that you presented, which upset numerous students who did not like him because of that, although they apparently respected him. Thus, Lionia Rozenbaum remembers that he and our classmate Sasha Kiriliuk loudly played cards at a physics lesson, which disrupted other students. Aleksei Yurievich asked them to leave the class and not come back. They happily did because they were bored by the lesson. They expected that Aleksei Yurievich would complain to the school administration and their parents. However, Aleksei Yurievich never did. Eventually, Lionia and Sasha became bored and missed the physics lessons that were, at times, fun for them. As a result, they came back and apologized to Aleksei Yurievich who let them back when they promised not to disrupt the class again. He told them that what you could do with your time and your attention was your business but you should not distract others from their learning choices.

At times, I was also upset at him, although I liked him a lot. He puzzled me and I was intrigued by him. Once I introduced his own position to him probably to please him or maybe in agreement with him. I expected that he would affirm that position, since it was his own and he was invested in it. To my big surprise, he crushed it as well. He possibly might not have a good memory and did not recognize his own points that he used to destroy my own position in the past. After that, I intentionally presented his own arguments to see how he would destroy them. I have learned a lot from him, particularly from his self-dialogue. I learned not to cling to my cherished positions and constantly seek the limitations of my own ideas.

At some point, my classmate Ira Gertseva and I decided that our physics teacher was a political dissident like one of Decembrists we both studied in our literature facultative. Ira remembers that Aleksei Yurievich looked very noble and ironic at the same time and was critical of the Soviet political reality. He also sounded very mysterious and foggy, full of many allusions. His judgments were strong and convincing. We sensed that he knew some very important secret and both of us wanted to be a part of it despite our fears. With trepidation, we came to him in his physics lab after classes were over and asked if he was a part of some secret political organization. He laughed but neither denied nor confirmed it. He told us that he would not talk politics with us because he wanted us to critically understand politics by ourselves without being influenced by anybody else. Nevertheless, he started giving us forbidden literature on art (including science fiction), politics, and religion (either published a long time ago or published by samizdat). He told us to be careful "in our sunny country" and not share this literature with people whom we could not trust, including our parents. Soon, I had a proof of his warning. My parents found the book he loaned, *The Master and Margarita* by Mikhail Bulgakov, which was semi-legal. They were scared and my dad threatened that he would complain to the school about the teacher. I learned the lesson and did a better job hiding these books from my parents. Three of us—Ira, Aleksei Yurievich, and I—discussed the books after classes at the physics lab. These discussions were very critical, as Aleksei Yurievich challenged any statement Ira and I made. However, no matter how upset we were when he crushed our ideas and opinions, we highly valued our small underground circle and we respected and loved our physics teacher. My relationship with Aleksei Yurievich continued long after my graduation from the school.

My classmate Aleksei Saverchenko, now a Russian diplomat, had a similar experience with Nikolai Nikolaevich Konstantinov:

> A reference to Konstantinov deserves a special mentioning—not with regard to math circles but with something else. I had many conversations with Nikolai Nikolaevich on, what I would call, para-political themes such as ruble inflation, emigration from the USSR, and so on, during our extracurricular hiking and KSP [the Club of Amateur Song] meetings in the woods. These conversations had an extremely important quality, which highly influenced my further life. They showed to me that there were other, different from official, but still legitimate and reasonable opinions. It was a sip of freedom in the ideological environment that did not tolerate alternative opinions. These alternative opinions shared by Konstantinov generated germs of doubts in my immature soul of a teenager. In my further life, these germs of doubt fully developed in me a strong conviction not to accept any given information without doubts but rather to test it against alternative sources, to analyze its underlining arguments, to draw inferences and conclusions by myself. All that was very useful in my professional and overall life, no less than knowledge and skills that I got in my math-school (email communication, February 28, 2017).

We had tremendously interesting intellectual peer environments—peer cultures—that went beyond math. In several interviews, Konstantinov highly appreciated the peer culture emerging in math classes. I will explore the peer culture of our math class—its bright and dark sides—in the following sections.

Gender Issues in the Math School No. 91

In the math class of the Moscow Math School No. 91 at graduation, we had 7 girls (24%) and 22 boys (76%) in our class. Out of probably a dozen long and short-term student teachers attending our class for 3 years of its existence, I do not remember having any female student teacher in our math specialized classes. In our conversation on this issue, Konstantinov explained this phenomenon by natural differences in gender destiny.

EM: Nikolai Nikolaevich, I remember that we only had boys—the undergraduates who taught at our school. I don't remember any girl.

NK:

There weren't many girls, of course, but there were some in various years.

EM: But why weren't there many girls?

NK: Well there were always a few girls in the math circles. During the entire time we had our math circles, starting in 1980, there were probably only three outstanding girls that entire time. In other words, strong girls who were as good as the boys.

EM: And why do you think things evolved that way?

NK: Well, I think that… You know, I had one strong female undergraduate, Ira Zhetvina, but I had her only in her first year. And later, when she was already in her later course years, and she and I would sometimes meet up at the conservatory—she loved music and knew music. We would meet quite regularly. Then she went into graduate studies after graduating from the university. Graduate study in the general physics department of the Energy Institute, MIEM. And, there she had a good adviser, a scholar; everything was going well, but later she and I met again at the conservatory, and she told me she had left graduate school and wouldn't study there anymore.

EM: Why?

NK: She said she had realized that being a theoretical physicist was not a female occupation. For example, a graduate student in that department, a theoretical physicist, works on some problem and he never knows whether he will be able to solve it. Now if a person works in some occupation other than a theoretical physicist, some other occupation, for example graduate study in some chemistry department, he has a pretty good notion that he will definitely be able to handle the topic he was given. But here, in the theoretical physics department, she said, I never know whether I will be able to solve it at some point or not. This, she said, is not a female occupation.

EM: Strange.

NK: Well, it seems strange to us men. But women do have a different destiny. One has to understand that a different

> purpose has been handed down to them from above. (Conversation between EM and NK, part 3, November 3, 2016)

For me, this sadly manifests sexism in Konstantinov. It is his limitation.

I interviewed my female classmate Alexandra Shlapentokh, who joined our math school only at the beginning of the 9th grade. I asked her about how she felt in our math class as a minority girl in a mostly male class. She said jokingly that she felt okay with being a gender minority in the class because of the greater attention from the males. Seriously, she replied that she felt super-comfortable. She experienced sexism only outside of the school with her past peers and with her parents who insisted that the profession of a mathematician was not for girls.

Alexandra's parents did not support her professional mathematician aspirations. They encouraged her to enroll in the math school in 9th grade because, according to Alexandra, they thought that the school was prestigious and it opened doors for respected technical professions for her daughter such as a computer programmer. (Note: a computer programmer in the USSR was viewed as both male and female profession at that time.) After Alexandra immigrated to the United States with her parents in the late 1970s, her parents insisted she pursue computer science and not a math major in her undergraduate college and then graduate school. It took her struggle with her parents (and, probably, with her own lack of confidence) to switch to mathematics later on and to become a professional mathematician, losing precious years for this struggle and transition.

As to Alexandra's peers, before she joined the No. 91 Math School, they often viewed Alexandra as "weird" and not "girly enough." Some of her friends, who were all girls, told her that she might get her brain swollen because she studied too much. It was very difficult for Alexandra to be herself with her peers before she joined the math school. She remembered that her classmates played a competitive popular game of wits of girls against boys in her school and the boys refused to play because Alexandra was in the girls' team. They seemed to consider that it was unfair game because the girls had an unfair advantage of having a too unusually intellectual girl. As a result, the teacher had to assign Alexandra a role of the judge of the game to solve the conflict.

Alexandra reported that she felt no sexism in our math class. There was no gender patronizing from the peers or our math student teachers (or the other teachers in the school). Nobody looked down on her because she was a girl. Nobody tried to discourage her study of math because studying math was not "a girly enterprise." Nobody was surprised when a girl in our class did excitingly well in math. Nobody expected that a girl would do worse than a boy in math or any other intellectual endeavor. In no interpersonal conflict that occasionally emerged among peers, was math ability gendered, nor used as a weapon. Alexandra referred to "the intellectual meritocracy" in our class where the peers stratified and rated each other according to math and other (physics, literature, music, poetry, dissident movement, and so on) intellectual achievements and creativity. Sometimes there were tough and even arguably mean and cocky competitions among some classmates but they were not gender based. However, arguably, this competitiveness, cockiness, and meanness might be gender-oriented in its own way. She remembered that once our general math teacher, Nina Iurievna Vaisman, assigned us a math problem, in which she obviously made a typo, making the problem much more difficult than she probably intended. When Alexandra noticed the typo, her initial response was to give up on trying to solve it and report the typo to the teacher. But then she considered that probably our classmate Yulik Baryshnikov would solve the problem anyway to prove that he was the best in our class. This thought made Alexandra persevere. And indeed, only she and Yulik solved the problem. All-in-all, Alexandra appreciated that tough, but fair, peer competitiveness that she experienced in our math class because she felt it prepared her well for challenges in her adult life. In short, she concluded that her 2 years in the math class were among the best years because there she was allowed to be herself and to flourish as herself.

Similarly, when I interviewed my former classmate Ira Kazakova (then Gertseva), she told me that she could not remember any sexism in our math class. Being interested in math less than some of her other classmates in her own judgment, Ira felt very comfortable in our class both in regard to our peers and in regard to our teachers. She remembers herself as being an informal leader of a girl group in our class. The only unpleasant gender-related incident she could recall happened in the 9th grade when a group of girls and boys and some of our math analysis teachers—college students (but not our main teachers)—went hiking near the Moscow woods in the late spring-early summer (I did not join any hikes or outside trips with my classmates). It was a very warm day and they decided to swim in a nearby creek. When they undressed to swim, one of the male college student made an unwelcomed sexual comment regarding Ira's body. She took offense, not only at the college student but also at our male classmates who did not stop him but rather joined him in laughing. The girls took Ira's side. As the conflict escalated, the girls left the group and returned home to Moscow by themselves. For the next hiking trip, the girls did not join the boys in our class. Later, all the involved boys and college students apologized to Ira and the rest of the girls. Their good relations were restored and there was no more gender tension. According to Ira Kazakova, both Venia and Andrei—our main math analysis teachers—while being only 4-year older than us were always professional and respectful to our female classmates.

In addition to the gender issues, I want to report that I could not remember any ethnic or socio-economic class tensions in our math class. My classmates, who I interviewed, did not report any of these tensions either. Also, many of my former classmates vividly remember a huge contrast between their prior schools with a lot of physical violence and physical bullying and our math class, where physical violence and bullying did not exist. That was my memory as well.

Dark Sides: Mean Competitiveness, Alienation, Verbal-relational Bullying, and Teacher Pranks

There was arguably a dark side in our math class. Some of us experienced and were involved in mean competitiveness, alienation, verbal-relational bullying, and teacher pranks.

Mean competitiveness involved rating peers according to their "math ability," "math brightness," "math genius," and assigning person-worth value to it. The proxy of this rating might include students' performance on the prestigious math Olympic games, solving particularly difficult math problems, offering an original and beautiful solution of a math problem, being mathematically ahead, beating peers in some math-related games, and so on. By itself, math competitiveness is not necessarily mean or bad—some students like it, some not. Respecting, admiring, and being proud of peers, who were mathematically talented and gifted, is very good in my view. This is how Konstantinov defined bad and good competition:

> NK: There is one fundamental point here: we don't have a first place, and in general there is no ranking. The certificate simply records what subjects you passed. It doesn't say what your ranking was.
>
> LB: When you've received a certificate, you're already a success, a winner. In the Lomonosov Tournament there are winners of the multidisciplinary competition, and that is something to be proud of… And as far as I know, the kids liked this very much. Especially the younger kids, because they get certificates, and they want to be recognized, and enhance their self-respect.
>
> NK: …. We follow this principle: there are schoolchildren who don't get any certificate, but they should definitely get something and not leave empty-handed. So we give out to everyone a report on last year's Lomonosov tournaments. There are no kids who leave the tournament empty-handed. As for the top prizes, there is a confirmed opinion about

this. They surveyed the children who got first prize at various Olympiads, and as a rule, when they grew up they said that it was unnecessary.

LB: Why?

NK: I'll give you an example. A girl in the 8th grade received the first prize in the Moscow math Olympiad. When she moved on to the 9th grade, she firmly intended to get first prize, and was very fearful that she would come in second. But she did win first. In the 10th grade she was already a complete bundle of nerves, got into the International Olympiad, and only got second prize. And she reacted to that second prize as a disgrace that would last her whole life.

LB: In the International Olympiad? A disgrace?

NK: She only came in second, and she was used to being first! You have to understand what kind of psyche we are dealing with. A very vulnerable psyche (Borusiak, 2010).

Journalist: Yes, I know that is one of your principles, you try not to traumatize the kids.

NK: No, it's not even… it's wrong to use the word "traumatize," We don't want to emphasize the competitive factor. So one of our activists actually proclaimed at a jury meeting: a math Olympiad is a competition not between people but a competition of people with eternity (Privalov, 2012).

NK: It is in the nature of children generally, to be drawn to competition. Say, a four-year-old boy says to a girl he doesn't know: let's see who can run to that lamppost first. They are happy to take part in such competitions. The Olympiad uses this natural gravitation to competition. The kids may not even know yet that they love math, but they find it interesting to compete. And, before you know it, they've gotten acquainted with math, and they like it. But one must remember that math is, after all, not a competition.

One of our activists, Sergei Markelov, is an amazing person who knows how to come up with totally surprising problems, and once he declared at a jury meeting: a math Olympiad is a holiday, so there are to be no sinuses! (a sine as a symbol of the school routine). and A math Olympiad is not a competition of people with each other, but a competition with eternity. We can compete with Archimedes: he proposed a problem, and we solve it. Here we are all together on the one side, and there is a lack of knowledge on the other. Knowledge competes with a lack of it and captures new positions. But the fact that people are also competing with one another is secondary. If the competitive aspect begins to come to the fore, and mathematics recedes to the background, then I think there is a distortion of objectives. It is incomparably easier to hold an Olympiad than to work with people systematically year after year and teach them something. But a major misalignment is taking place (Gubailovskii and Kostinskii, 2004).

This is why treating peers as more or less worthy based on this math competitiveness is rather problematic, in my view. Looking down or making mean, humiliating remarks about mathematically "non-brilliant," "non-genius," "giftless" ("bezdar'" in Russian) students, is even worse. The mean competitiveness was not omnipresent in our class but was limited to a small group of peers. It was counter-weighted by the fact that my peers had diverse talents outside of math, making the mean math competitiveness rather limited and contained. Some of my peers played music (e.g., Asia Belaga), some wrote poetry (e.g., Oleg Glushko), some wrote thriller stories (e.g., Aleksei Zheliaev), some were involved in a dissident movement (e.g., Volodia Muzykovsky and Nikolai Ukhanov), some had athletic gifts (e.g., Pasha Shaburov), some had a gift of kindness (e.g., Ira Segal', Ira Gertseva, and Nadia Kas'ianova), and so on.

The counter-currents of egalitarianism and mutual respect were also very high. Thus, my classmate Aleksei Riabinin reported in his interview that in comparison to his previous experience in a Moscow school that specialized in the study of English, our math class was very respectful to each other and not mean at all. He remembers that in his English classes of the previous school, peer physical and verbal meanness, mean gossiping, and scapegoating were omnipresent and he regretted that he participated in it. In our class, he felt he was liberated from it. Unfortunately, I could only partially agree with him. My past school also involved a lot of physical, verbal, relational, and even sexual peer meanness, abuse, and bullying, although I did not experience any of that in my math class. However, mean competitiveness was present in our math class. I do not remember our teachers ever supporting it. When I interviewed Ira Kazakova (formerly Gertseva), she made an interesting hypothesis that this mean competitiveness might come from the parents of some of our classmates.

At the end of our graduation, in the 10th grade, a few of our classmates—Ira Gertseva, Oleg Kazakov, and Aleksei Riabinin—conducted a survey about how we felt about our math class and about our future aspirations. To my surprise one of my classmates reported alienation and a sense of being unwelcomed in the class.[24] In our class, we had a, more or less, stable group of peers who attended many events together, invited each other to parties, which often involved alcohol intoxication, and intense interactions. I was on the periphery of this group, attending some events and parties but not others. Some other students were out of this group and had their own friendships. Some students in our class seemed to be loners. I do not know how much alienation was in our math class and how much math competitiveness and specifically mean math competitiveness contributed to that, if at all.

In our math class, we had a girl who was verbally and relationally bullied by a group of some of my classmates. Often they were saying mean things to her face or about her. I could not understand why. My classmate Oleg Kazakov suggested that two of our classmates who had been her classmates before our math class brought on the animosity; they brought this animosity from the past. However, it was unclear for us why we did not stop it in our class. In one of my interviews, I asked another classmate why some of our classmates were so mean to her. He replied, “She constantly asked bizarre questions. For example, we were going somewhere together and she would ask, ‘Look at this bus. Imagine it is full of children rolling at full speed down a hill without a driver. How should one jump under the bus so it would stop?' As a result, this question fully characterized the girl. I think that the person with such sacrificing attitude asks to be a victim [chuckles]." I always felt sorry for her and I did not know how to help her other than offering her my friendship. We were friends for a while but it did not work well in the long run, probably, because pity rarely is a good base for genuine friendship. I wish I could relate better to her being different. I wish my classmates were more open for people who were different and who could ask "bizarre questions." Alexey Saverchenko also remembers one of our classmates being verbally and relationally bullied by consistently being called "pimple" by some of our male classmates. I do not remember that.

It seems to me that a part of all Soviet schools' unintended, hidden curriculum was to teach students' animosity to at least some of their teachers, as resistance to the teachers' agentic imposition of authority. I have noticed that when Soviet school alumni meet, one of the popular topics is often a collective resistance, badgering, or even active humiliation of hated and unloved teachers. Usually, the reasons for students' collective attacks on their teachers could be the teachers' dullness, feeble-mindedness, injustice, bad temper, insensitivity, poor pedagogical quality, being boring, ill-intent, prejudice, and so on, perceived and/or experienced by the students. In my personal recollection, two of my high school teachers were the special targets of our collective attacks. First was the teacher of Civil Defense. Despite his seriously looking uniform of a military officer, a Major, he was pretty harmless and perceived by us as rather stupid, dull, boring, and pedagogically inept. He delivered rather plain lectures about "American Imperialists and Militarists" and constantly threatened us with the Nuclear Holocaust. Being math and science savvy in our math specialized high school, we entertained ourselves by asking him technical and scientific questions, on which he did not know answers but did not want to admit it and provided us with ideological clichés that made us laugh. But we also made more serious pranks on him. Once, my male classmate brought an alarm clock and hid it under the teacher's desk. At the beginning of a regular boring lesson, we informed the teacher that American Imperialists placed a bomb in our class. Of course, he did not believe us. At some point of the lesson we started counting backward from ten to zero, publicly announcing the explosion. But nothing happened (except our collective laughter). In ten minutes, we repeated the performance but this time the clock alarm went on. The teacher was rather scared as we theatrically and cruelly built the cathartic moment. On another occasion, some of my classmates launched a self-made black-powder jet rocket in his class. The rocket did not take off but fiercely danced on the floor exhausting a huge stream of black smoke. The Civil Defense teacher tried to put it down by stepping on the rocket, which was not easy because the rocket unpredictably moved. When he managed to step on the rocket, and then raised his foot, it moved up with new fury because, in contrast to a typical fire, black powder produced oxygen while burning and could not be extinguished when its access to external air was eliminated. We all knew that fact and the teacher's helplessness—interpreted as stupidity—entertained us a lot that day. We laughed hard until tears formed, although some girls in our class thought that it was too much humiliation and they had pity on the teacher; however, their compassion evaporated when he went to complain to the school administration.

Of course, we were often punished for our teacher-humiliation pranks by the school administration. However, we had an unwritten 3N rule: Never report on classmates who made a prank; Never reveal his/her name under school administration interrogations and threats; and Never blame the prank ring-leader for a collective punishment, however, harsh it was. Reporting to the authority was a strong social taboo that was probably deeply rooted in the Soviet historical memory of the Stalinist purges.

Konstantinov's Math Pedagogy Influence after Graduation from Konstantinov's Math School

After graduation from our Math School No. 91, many of my classmates and I tried to enter the Mechanico-Mathematical (Mekhmat) Department of the Moscow State University (MSU).

Some of these classmates were ethnically Jews, like me, and some Russians. Jews knew that because of the state Antisemitism, they did not have a chance to get into MSU—Jewish applicants were failed during the MSU entrance examinations regardless of their performance. My parents tried to dissuade me from applying to Mekhmat but, like some of my Jewish classmates, I decided to try to make sure that indeed I did not have any chance. I expected to fail a Russian essay exam because literature and composition were subjective and ideological and thus easy to manipulate. The oral math exam was also a possibility because it did not leave paper traces (Shifman, 2005). However, I was surprised I failed a WRITTEN math exam, for math is an "exact" science and a written exam leaves paper traces. During my appeal, I was interrogated and humiliated by MSU officers specializing in keeping Jews out. I had "impure pluses" on all 5 math problems: plus-minus, plus-dot, minus-plus (for the last problem that I knew I had not solved). "Impure pluses" meant that a problem was not solved completely and was not counted (Shifman, 2005). Let me provide one example of a math problem that was scored as "plus-minus."

It was a geometric problem involving a right-angled triangle. I wrote in my solution that, "According to the Pythagorean Theorem, the square of a right triangle's leg is equal to the square of hypotenuse minus the square of the other leg." The examiner crossed out my wording "according to" and replaced it with "from." My two interrogators at the appeal told me that: 1) I did not know the correct formulation of the Pythagorean Theorem and 2) I was not skillful in Russian enough to understand a sematic difference between "according to" and "from." I remember a Jewish girl at a neighboring desk suddenly jumped from her place (sitting like me between two of her big guys) and yelled at them, "You will live in this shitty country, while I'll study at Sorbonne!" She then stormed out of the room. I think she was right about the racist Soviet Union being a "shitty country" and I hope that she managed to emigrate from the USSR[25] to France and her dream of studying in Sorbonne became true. All my Jewish classmates failed and all my Russian peers succeeded (see, Frenkel, 2013a; Shifman, 2005, for description and analysis of these systematic Anti-Semitic practices at MSU sanctioned by the Soviet State).

I went to the Moscow Automobile-Road Institute (MADI), the college that accepted Jews, to major in computer science. My older brother and two cousins graduated from it. Both my teachers Venia Dardyk and Andrei Pechkovksii were senior students there. Venia told me that when he and some other kids from math schools, many but not all Jews, failed to get in MSU in 1973, Konstantinov advised them to go to apply to MADI. When they all were accepted to MADI (the same computer science department), Konstantinov talked with the MADI's math department to excuse these group of the students from attending math courses. Instead Konstantinov organized informal math classes for them inviting his friends who were math professors from MSU. Unfortunately, these informal math classes existed only one year because, as Venia told me, the class curricula were over the top of many of the students. Despite their informal nature, the class instruction was traditional lecture-based. In my interview with him, Konstantinov concluded that his pedagogical experiment of creating informal undergraduate math classes for students from math schools in MADI failed. However, he was only partially right.

In the fall 1977, after we graduated and joined our colleges, our math analysis teachers Venia, Andrei, and their college friends organized an informal math club focusing on artificial intelligence (AI). We met every Saturday night, after our formal college classes to discuss AI. Thus, Yuliy Baryshnikov remembers this regarding our AI club, "Regarding Markov's substitutions, I remember how we had a lengthy discussion with some friend of Venia and Andrei (while serious planning to work on this) about how to design an artificial intellect. The idea was to take random substitutes (objects) and practice until a brick from the roof fell firmly to the ground (simulating gravity). Before even 40 years elapsed, deep learning almost turned our fairy tales into reality...." After this math club dealing with AI, my MADI college classmate Matvey Sokolovsky, an alumnus of Math School No. 57, and I organized a math circle for our MADI peers who were interested in math. We invited a MADI math professor who involved us in interesting applied math problems of automatization of street traffic lights for more efficient traffic. At the same time, Matvey and I petitioned the MADI administration and achieved (with constant struggle) free attendance, similar to Konstantinov's request a few years earlier. We used our time to attend math lectures at Mekhmat MSU. Some MSU math professors, such as Nobel-prize winner Vladimir Arnold, were very sympathetic and supportive to young Jews interested in math.

In 1978, in my second year at MADI, I was invited by one of my math classmates to attend an underground university, the Jewish People's University (JPU), taught by famous Soviet professors from MSU and other prestigious colleges (including some Russian and non-Jewish professors) who heroically resisted the Soviet State-sponsored Anti-Semitism. However, in July 1982, the KGB cracked down on this underground university for Jewish students and one organizer, Bella Abramovna Subbovtskaya, [26] was apparently assassinated by the KGB in the fall of 1982 (Karp and Vogeli, 2010, pp. 207 – 10; Szpiro, 2007). The other two organizers of JPU Valery Senderov, a math teacher in the Moscow math school 2, and Boris Kanevsky were arrested, imprisoned, and then exiled (Shifman, 2005). I attended JPU for 2 years. In 1979, my interests shifted from math and physics to philosophy, psychology, and pedagogy. Later, after the collapse of the USSR in the early 1990s, Nikolai N. Konstantinov organized his own university, The Independent University of Moscow (Ilyashenko and Sossinsky, 2010).

Many (but not all!) of us have remained closely connected with each other and some of our teachers, and other alumni and teachers of our math schools. The math school created long-term networks, communities, and cultures of alumni and teachers. Some alumni got married to each other and created families. As a son (Lionia Rozenbaum) of my classmate (Zhenya Emelin) commented on his father's classmates/friends, "You're weird. When you get together and become drunk, you start discussing math problems" (conversation with Lionia Rozenbaum, December 7, 2016). Many of their children, themselves, went to math schools. Many math school alumni became professional mathematicians—pure or applied. Konstantinov remembers, "I remember how a group [of math-school alumni] once got together. There were a lot of unfamiliar people there, and one guy was asked where he had graduated. He began to respond about which institute [college] he had attended. 'No, no! What [math] school did you graduate from?' It didn't matter what institute [i.e., college/university] he had graduated from. That was the attitude" (Borusiak, 2010). Alexei Riabinin reported to me that the main thing he learned in our Math School No. 91 was to love people.

Part II: Educational Philosophy of Konstantinov's Authorial Math Pedagogy

In my analysis, Konstantinov's innovative authorial pedagogy involved at least four major related transformations of conventional pedagogy:

1. *The purpose of education*: Moving away from socialization of the students in the math practice as the goal of education to promoting "wings" in the students: their self-realization, self-actualization, self-inspiration, which may or may not occur in math.
2. *Guidance*: Moving away from teachers' lectures and students' exercises as the main form of guidance to students solving new problems and teachers testing the students' solutions with them. In short, using Bakhtin's terminology, it is a move from math as "the authoritative discourse" to math as "the internally persuasive discourse" (Bakhtin, 1991; Matusov and von Duyke, 2010).
3. *Learning environment*: Moving away from forced assignments supported by summative assessment (ranking grades) to a free-choice learning environment, in which the students are free to engage or not to engage in targeted activities and free to choose problems to engage with. Essentially, it is a move from teaching as the main form of guidance to support of the students' autodidactic learning.
4. *Pedagogy*: Moving away from technological pedagogy based on pedagogical techniques and strategies guaranteeing the students' learning of the preset curriculum, to authorial pedagogy, based on the teachers' pedagogical and students' self-studying authorship.

The Purpose of Education

Answering a question regarding the mathematical level that his students achieve, A. Leman (Konstantinov's follower in teaching in one of math schools), said: "We don't teach people to be mathematicians—we teach them to be free" (Ilyashenko and Sossinsky, 2010, p. 38). When in our conversation, I asked Konstantinov about the purpose and main value of his authorial pedagogy, he hesitated with an answer. I suggested to him I could provide my own answer to inspire him and he liked the idea. I talked about promoting students' authorship, both mathematical and otherwise authorship. Konstantinov listened attentively and then interrupted me with another metaphor. Using a popular colloquial Russian saying, he said as an educator he was interested in "people with wings." It seems to me that this Russian metaphor might go back to Ancient Greek, to its myth of Icarus, who built wings to fly to the sun. In any case, the Russian metaphoric saying refers to people's self-realization, self-actualization, self-inspiration, creativity, and transcendence of the people's being, nature, and culture, as the given. This self-realization does not need to be in math but can be in other areas of human activity, either intellectual or not intellectual, according to Konstantinov. He provided several examples of "non-math wings" in his math school students that he appreciated:

NK: [We had one fellow in math class], an expert in everything, but abstract math did not interest him. That's the way he was. So when he graduated from school, he enrolled in an institute [i.e., a college], and he was attending the Bauman Institute.[27] But he did something very unusual. His father was also a jack of all trades. He set up a dacha outside Moscow so nicely that it was no worse than their Moscow apartment. So when these people moved to Moscow by September 1st, as usual, this fellow Timofei said that he didn't want to go to Moscow, but would rather stay at the dacha. His parents did not object, and he lived by himself at the dacha. He boiled potatoes for himself, but enjoyed total freedom. He found an abandoned, broken-down car at some dump. He repaired it and now drives it around. He also developed a warm friendship with the local police. First of all, all of his papers are in order. Second, if a police officer has a car problem, and if Timofei is somewhere nearby, he will fix it right away. So, the police treat him with great respect. And now he found a second car, also abandoned. He has already repaired it and is now switching to that one. You see, his lifestyle is not like that of conventional mathematicians. Anyway, the fact that he did not study math analysis did not cause his instructor to try to get him expelled from the school. He [his instructor] tolerated it. Because this was a very special person, on a very high level. And I have a couple of other examples.

We also had a pupil, he was in my class, by the way, who also didn't solve a single problem in analysis. But he took first place in Moscow in his knowledge of Spanish, and now he is helping to establish contact with Spanish-speaking countries. So, should he have been kicked out or not?

EM: Actually, it would be interesting for me to find out what your values are. If math is not important, then what is? How would you define that?

NK: I don't know; I just resolve that question in a concrete situation. But for me to have some theory in that regard, I don't have one yet.

EM: Let me suggest something from what I am hearing, and you tell me if that is the case or not.

NK: Give it a try.

EM: It seems to me that you focus on the author. It may be an author in math or in some applied matter when something is done manually. Or authorship in language. You're just interested in authorship. English has the word "agency," and Russian has a word that's close, "author." Alternatively, maybe "talent."

NK: Well, I would use another word. So, we got to know a person who came to work for us. A very nice fellow, everything was fine. But later, when we tried to evaluate the person, I said I liked everything about him, but I don't see wings on him. So the word "wings," in my opinion, is more apt.

EM: In other words, a "person with wings!" That is a kind of creative authorship. A creative person with initiative.

NK: Yes, something like that.

EM: Thank you, that's a very good metaphor.

NK: Can I give you another example? A very talented person. He has already graduated from university, and he is already teaching in school. Here is what is amazing about this fellow. He is an amazing workaholic; he works so much, you can't imagine. It's impossible to learn that from me. Here's what he liked: he selects problems for middle-grade pupils, for example, from the 3rd through 7th. And he has begun to put out a journal—you probably know the journal Kvant? Well, he created a kind of younger brother to *Kvant*[28]—the journal *Kvantik*.[29] He publishes it, it is on sale at kiosks, and more than just middle-grade pupils are interested in it. Retirees also like to read it. Anyway, he spends a great deal of time on *Kvantik*. And when I once started talking among my friends about wings—what I was just telling you—this person, Sergei, said: "If I didn't have wings, do you think I would be occupied with publishing this journal *Kvantik*?" That is absolutely right. Thanks to having wings, he spends a ton of time on putting out Kvantik.

At the end of the day, a fully developed person with wings is one who can self-inspire him/herself for initiation of new creative endeavors, projects, and tasks. Nurturing such qualities involves a special pedagogical orientation. Konstantinov values students who love to study, passionately love math (or another endeavor), and who can persevere in spite of their frustration and occasional failures in pursuing their own interests and passions (rather than A-students or those who are winners of a prestigious math competition). It seems to me that Konstantinov's notion of the ideal student was realized in the selection of students for math schools. All students who did not drop and came to the final sixth part of the examination were accepted regardless of the examination results, that is, how many math problems the prospective candidate solved or did not solve. The test was not about the number of solved problems, but about whether the student's interest in math could survive the frustration of many math problems not solved. An ideal student is one who wants to study and who can "persistently think about an unsolved problem."

> NK: Indeed, there are not enough capable youngsters. However, I think the problem is not only that it is impossible to teach everyone; it is impossible that a person wants very much to learn. There is a well-known physics teacher who conducts a seminar for adults, which is attended by physicists and some teachers. He once posed a question to them: "Which pupils would you like to work with—with honors students and winners of Olympiads, and who else?" And nearly everyone gave roughly the same answer: it isn't important whether he is an honors student or an Olympiad winner; only one factor matters: whether the person is capable of relentlessly focusing on an unsolved problem. Because sometimes a person is very capable, but he lacks that relentlessness. Then nothing will pan out, nothing. It is important that the interest in this grows throughout his childhood. But sometimes this interest is killed in school; for example, they instill a fear of math in people (Borusiak, 2010).

For Konstantinov, an ideal student seems a combination of being a dilettante (from an Italian word translated as "delicious"), who loves to engage in some endeavor (e.g., math) and loves to study it, and also one who can persist through frustration and failures often associated with self-study and solving math problems (cf. Anderson, 2010; Matusov and Brobst, 2013).

How Can an Educator Support Education for Wings?

I asked Konstantinov how to support "a person with wings" and promote development of his/her "wings." Konstantinov replied that there is no recipe for that but the educator him or herself must have wings. However, there are pedagogical and organizational conditions that facilitate or hinder the process of developing "wings."

EM: Suppose some educator wants to support children with wings. How can one help people by supporting the development of their wings?

NK: It seems to me that you have to be just as good yourself—that is the hardest part. I don't think there is any algorithm here. Such a person is always different; how can there be an algorithm here? Clearly, you have to be different yourself.

EM: There was a philosopher in Russia named Vladimir Bibler, he called this "a person of culture" (Berlyand, 2009; Bibler, 2009). That is a person with wings who does something that no one else does and wants to do it very much. He does it not because it can be sold, but because he can't live any other way. To get back to our conversation about the pedagogical institute, what kills those wings in school are the constant, strict rules: these assignments have to be done by Friday; otherwise you will get a failing grade. Do you agree with this?

NK: I agree. When we were first coming up with the idea of starting math classes, Kronrod,[30] who was one of the initiators, thought that all this would be decided by the regional [Communist] party committee, although it turned out later that the regional party committee had nothing to do with it. So he arranged with the regional party committee that we would operate in general without grades. But this didn't pan out, it didn't work out. But there was such an idea: what are grades needed for? You understand yourself that one can live without grades.

EM: Incidentally, I wanted to bounce off you another observation of mine. Venia and Andrei simply gave everyone "fives" at the end.

NK: Right. They did that precisely for the purpose of, in effect, abolishing grades.

EM: Okay, so aside from the fact that we had passionate teachers like Venia and Andrei saying grades were unnecessary, assignments hanging over you were harmful, what else did you discover, what other things were harmful to the development of wings and what things were important for developing wings?

Narrow-mindedness about subjects: what you told me about the children who did not necessarily study math. It's also narrow-minded when you look at one thing and forget that the world is richer than math or something else. That is already a third principle. What else did you notice?

NK: You know, a major process of bureaucratization is now taking place in our school [involving rules rather than authorial judgments by educators] (Conversation between EM and NK, part 1, October 30, 2016).

From this conversation and other sources, I can abstract the following principles of Konstantinov's authorial pedagogy to support people with wings:

1. The educator him or herself must have strong wings.
2. There should not be pressure on students that distract them from their interests, excitements, and inspirations (i.e., "wings") by the creation of the pedagogical regime of survival.
3. The educator should avoid or sabotage grades that constantly create pressure.
4. The educator should avoid the regime of assignments that colonize students, and distract them from what they want and are excited to do.
5. The educator should avoid bureaucratization and avoid being guided by rules. Instead, the educator should be guided by authorial judgments of what is good and what is bad in each particular situation and by taking personal responsibility for the educator's own risky decisions.
6. The educator should avoid seeing the student narrowly and not letting the students develop wings in a broader or even entirely different field.
7. The educator should involve the student into setting their own problems and goals.

Preferring Working with Older Children

Konstantinov insisted that his ideal students should have certain voluntary commitment to certain field of practice (e.g., math) for his education for wings to develop their own authorial voice in this practice. For this, students' personality should be mature enough and often starts with adolescence. Young students' wings have the holistic, broad, non-differentiated, and non-committed nature.

NK: I can only say why I don't feel like working with little kids. . . .

I was once asked about this by Sergei Fomin [an organizer of math circles and Olympiads]: "Why don't you work with the 5th or 6th grade? They are such appreciative youngsters; they take in everything so well." I gave him this response: "When I have pupils in the 10th grade, then even if they don't know something, they are in any case people who have their own system, their own scale of values in life. But when I work with a 5th-grader, at best I see my own reflected scale of values. And so I don't find him very interesting."

Take a historical analogy. When Beethoven began studying under Haydn, the latter simply could not understand what was interesting about that pupil. He duplicated exactly what Haydn did. But when Beethoven showed something of his own, that is when Haydn took an interest. And he realized that it was very interesting.

Little kids can be drilled into shape. But there is another aspect: a person should develop broad interests. If he doesn't have them, what will he do? I will give you an example. I was once talked into taking a 6th-grader to Estonia [to a summer math camp]. He made an effort there, he was a librarian, and he even attended a combinatorial-analysis circle. But it was clear that his mind had not matured. He couldn't understand some simple things. Then, he began to understand them, entered School No. 57, graduated from it successfully, and everything seemed to be fine. However, after school, he did not apply anywhere, he was already 30 years old and, since then, he has not studied or worked anywhere, and he is fully satisfied with that kind of life. He doesn't need anything. Yet his parents did all they could so that he would develop as rapidly as possible.…

It seems to me that when a person is taken out of his natural milieu, he may lose more than he gains.

Interviewer: But circles for the 6th grade are something different. It's not the same as a 6th-grader going to a 10th-grade circle.

NK: V.L. Gutenmacher [a popularizer of math, now living in the United States] began to lead a geometry circle there for little children when his daughter was attending School No. 91. I asked him: "What do you do there?" And he responded: "Of course, the notion that we study geometry there is a relative term. At the first session we discussed the question: What do the words *tsirkul'* [compass] and *tsirk* [circus] have in common?" I think that is actually the right thing. The smaller the people, the more diffuse their interests are. A child may be writing a formula, and he will take an interest in the pencil he is using to write the formula. There is no point in trying to conduct a concentrated course for them. But such sessions where there is a discussion of what is in common between a *tsirkul'* and a *tsirk* are very appropriate for that age (Konstantinov et al., 2002, pp. 47 – 48).

EM: I read your very interesting reflections about why you don't like to work with younger schoolchildren. Such as why you prefer to work with the 7th grade rather than the 3rd or 4th grade. I don't know when the interview that I read took place. Can you add anything to it?

NK: Well, it seems to me that they need a somewhat different education. It should be less narrow.

EM: More integrated? More holistic?

NK: It should have the most varied things in it: why a cat meows, why a fly has six legs, or why a bulb burns out. In short, a very broad range of questions that children study in a cursory manner. Studying something in greater depth is for older children (Conversation between EM and NK, part 3, November 3, 2016).

To me, Konstantinov's position regarding younger students suggests that their curriculum should be even more open and emergent than the curriculum for older students. The K-6 curriculum probably should not be thematically defined by a discipline but should allow free multidisciplinary inquiries. It prevents students from early disciplinary commitments to which the young students often are not ready (cf. Lobok, 2012).

Teacher's Guidance: Testing Students' Solutions of Math Problems over Lecturing

One major change that Konstantinov made early on was to move away from lecturing and exercises as the main form of guidance. Instead, the main form of guidance became students' solving math problems and teachers' testing their solutions with the students. Math problems are different from exercises because math problems aim at the introduction of new, emergent concepts and ideas; whereas, traditional exercises involve the application of the concepts and ideas that the students learn through lecturing. For example, in a conventional school, a teacher introduces a new theorem and then proves it in front of the students. The students have to understand and memorize the theorem and then use the assigned exercises to correctly apply the theorem. Finally, the teacher checks the students' correct understanding of the targeted theorem through oral and written math tests: reproduction of the theorem's proof and its application.

NK: When I assign problems, I hand out leaflets to everyone with the condition that the person see the written text. That is better than if I write something on the chalkboard and they copy it. . .. In general, there is a presumption in our country that if a lecture has been delivered, the undergraduates have mastered it. But the lecture system is ineffective from the outset because some people write things down quickly but don't understand anything, while others are distracted. And people are going to study all this only before the exams. There is also a misconception that first you should fill people's heads with knowledge and only later they should apply that knowledge. A person simply stops taking things in. At the Higher School of Economics, the math department tried to repair this error. There, someone delivers lectures, and they are immediately posted on the Internet. In addition, every student there regularly delivers solutions to problems. As he absorbs the material, it immediately becomes a workhorse (Leenson, 2012).

In contrast, in Konstantinov's pedagogy, students are often faced with a new theorem as a form of an original math problem that they voluntarily approach to solve solo or in collaboration with other students. The students may or may not successfully solve this problem/theorem on their own. When a student or a group of the students feel that they have solved the problem/theorem, they report it to a math analysis teacher who will "accept" the student's solution by: listening to the students' solution carefully; asking probing questions; attracting the students' attention to the most interesting, creative, original, and beautiful features of their solution; and providing guidance when the students get stuck. The math analysis teachers' guidance is often provided in response to difficulties that the students experience attempting to solve the problem on his/her/their own. When a student successfully solves a problem/theorem, it becomes a new tool ("workhorse") for solving other problems/theorems, which makes a problem a theorem. By "theorem," I do not mean the classical theorem listed in the traditional math textbooks, but any problem that can be become a tool for solving other problems. Facing a new math problem/theorem provokes a new interest in the student, which leads him or her to a deepening interest in it.

NK: …the natural course of learning in math is such that a person initially becomes interested in a problem, and then he somehow begins to delve into it. However, at lectures, it is the other way around: you haven't even had time to find out what is interesting about a problem before you are already provided a solution. A theorem is being read,

> but you don't know what it is for, and then you have to learn it, then another one, and another one, and so on for five years. But that is already wrong (Privalov, 2012).

In Konstantinov's pedagogy, math is always authorial, discursive, and "internally persuasive" (cf. Bakhtin, 1991; Matusov and von Duyke, 2010). The math truth is established, defined, and controlled not by the authority of the teacher or textbook or test, but the student him/herself through a persuasive discourse in a math community, involving the peers and the teachers. Of course, lectures also play a role in his pedagogy, but the lectures were often subordinated to the primary process of solving and reporting/testing the math problems.

EM: Nikolai Nikolaevich, I would like to ask you now about the origins, and I have read a lot already in your interviews about the origins of authorial pedagogy. I knew that these math circles existed even before you, but from what I've read, they were, by and large, different. They were more lecture-oriented. Is my understanding correct?

NK: They were different, of course. They were not entirely devoted to problem-solving.

EM: Could you explain why you switched from that lecture-oriented form or lecture-and-debate form to problem-solving and the discussion of problems?

NK: I think the traditions of the MekhMat [a math-physics department at the Moscow State University, MSU] were the main factor here. I had a friend with whom I shared a desk, and his mother was a biology teacher. And since she was a teacher, even in biology, she gave us a voucher to attend the circles at the university. And the circles had lectures, first of all. The first lecture I attended was a lecture by Yaglom. Yaglom delivered a lecture on induction in geometry. There were some little geometry problems in which you had to find out how many parts a diagonal divides into a polygon; quantitative problems solved by induction. Anyway, do you know what stunned me at that lecture? Yaglom was lecturing and sometimes posing questions to the schoolchildren sitting in the first row. In other words, he was standing at the chalkboard, and schoolchildren were sitting in front of him and listening. And so he says, addressing the hall: "Is there a theorem like that?" And he formulates some theorem. And I was in a university for the first time and thinking, "This is what we have come to—a professor is lecturing at a university and doesn't know what theorems there are." He asks the schoolchildren whether such a theorem exists. "Well," I thought, "he's a professor, and doesn't know theorems." I reacted, of course, in a narrow-minded, scholastic way: he's a professor who should know everything. And suddenly he says, "You don't remember whether there is such a theorem or not? Well, it doesn't matter, we will prove it now." And I was completely stunned. He doesn't know whether such a theorem exists, but he doesn't even need to know this. If necessary, he will prove it himself, and that's it. In other words, there was a sense that we were living in an altogether different world. In school you had to memorize everything to know it. But here, it turned out, you could get by without memorizing something and still know it.

EM: In other words, knowledge here comes from re-production. If I need to know something, I will relearn by deriving it, and not by recalling it.

NK: It is not me memorizing everything in advance and then making use of it. Of course, it's a completely different approach.

EM: In other words, a scholastic approach, a good scholastic one, is first to understand and memorize. And then to reproduce your understanding—what you memorized. Here it is completely different; here it is problems every time, interesting problems, which are solved. So if you have forgotten it, it doesn't matter. It is a new problem that has to be solved again now.

NK: So I had this strange experience in the 1st grade. We were told to memorize the multiplication table. Now this is

> done in the 2nd grade. At that time it was in the 1st grade. So the teacher calls on Ivanov. Three times three. He stands up and answers: nine. And she gradually questioned everyone this way. I realized that at some point my turn would come. But I hadn't memorized the entire table. She asked me, six times seven. Six times seven was, after all, a relatively difficult one. And I hadn't memorized it and thought, what do I do? I immediately stand up and figure it out—"six times six is thirty-six"—that I already knew. "And six times seven, you need to add another six." While I was standing up, I figured it out and answered correctly. And afterward I was convinced that I had fooled the teacher. We had been ordered to memorize and I hadn't memorized. I knew the answer, but I hadn't memorized it. In other words, I hadn't carried out her order. The funniest thing is that, to this day, whenever I have to multiply six times seven, I do that operation (Conversation between EM and NK, part 3, November 3, 2016).

A move away from lecturing to listening and testing students' own solution of math problems-theorems is a move from, what Bakhtin (1991) defined as "the authoritative discourse" to "the internally persuasive discourse." In the latter, both the student and the teacher are members of a discursive community, in which both of them have equal rights to challenge and to reply to any idea in the discourse. Truth is established in this discourse only after a student can prove it against all possible questions, objections, and challenges from equal members of the community. The teacher does not have more epistemological or other authority in this process. When the student's math discourse has survived the teacher's (and peers') questions, objections, and challenges, it becomes a solution and a proof (cf. Latour, 1987). In contrast, in conventional lecture-exercise pedagogy, the truth is controlled by the authority of the teacher. The goal of the student is to understand, accept, and reproduce this truth.

> NK: That is how, little by little, I got involved with math circles, and by the fifth year, I had become so brazen that I organized a seminar for first-year students in the MekhMat. And then I grasped something amazing, which was absolutely new to me. The level of mutual understanding between instructor and student becomes completely different when the instructor is accepting problems. This systematic work, when I am trying to understand your thought, and you are trying to understand my thought, is a completely different level of mutual understanding than the one that occurs when a lecturer is lecturing. And especially now—remotely… someone sitting at home can listen to a lecture at a university… that level will not be there. It inevitably turns out superficial. He can't even ask a question (Dorichenko, 2010).

Intimate interaction between the teacher and the student in Konstantinov's classroom is not limited to "accepting the student's solution" but also to challenging the student with new problems. Again, the challenge was among equal people in a discursive community, who take each other ideas seriously and without any patronizing or nonsense. The teacher helps a student move from intuitive germs of ideas to the development of the full scale understanding:

> NK: …I had a pupil in school, and now he's an undergraduate. But when he was still in school, I began asking him questions like this: "Think of a definition for the exact upper bound of a set." And he successfully came up with one: it's a number that is greater than or equal to any element of the given set, and it cannot be reduced. Later, when he was in the 8th grade and had not yet learned the definition of a limit, I began asking him questions like "Try to come up with a definition for a limit." I gave him about ten versions of the definition, and he resolved all of them on his own. And in certain cases he would say, "No, that's wrong, because…" But I was astonished that this person came up with all the definitions virtually by

himself. They are in fact practically identical. He's a very capable fellow, and now he is attending a university.

EM: And what was it, if you remember, that spurred you to give him the problem of coming up with a definition? In this case, with this pupil?

NK: I will say that usually people start blabbing some nonsense. For example, when I was in about the 7th grade, a classmate asked me—and this was a regular class as there were no math classes at the time—"Do you know higher math?" This was a completely unexpected question for me, and I said "no." So he says to me, "Look. A value is called infinitesimally small if it continuously approaches zero but never reaches it." I took in this assertion as a very wise one. Good heavens, something like that had never even occurred to me. I had gotten some information that there is this higher math, but it had come to me in some odd manner. But when a person grasps some essence, he begins unerringly to create all these definitions, now that he has grasped what it is all about. It's terrific when a person has already intuitively grasped everything and begins to formulate correctly.

EM: In other words, when you see that a person is beginning intuitively to understand, the next task for him is to formulate, that is, to find a form for it.

NK: Yes, to find a form. But if he knows the form in one case, this helps him to find the form in other cases. (Conversation between EM and NK, part 2, November 11, 2016)

Finally, Konstantinov defined three major pedagogical principles for math classes, which in my view constitute the principles of "the internally persuasive discourse" (Bakhtin, 1991; Matusov and von Duyke, 2010):

> The main principles of working in math classes are to be *thoroughness, deliberateness*, and *independence*.… Thoroughness means that a topic is not covered in a tentative way ("you will be taught this properly at the university") but

definitively (which does not preclude later returning to the topic at a new level). A loss of thoroughness leads to a loss of interest. A pupil who did not completely understand something once and did not completely understand something another time contaminates his studies, in the end, to the point where they start to repel him. Conversely, thoroughness makes it possible to always find something new in ordinary things to be interested in. The teacher's key role is not to describe and explain but to thoroughly check and analyze any errors, while retaining a sincere interest in all of the pupil's successes. This interest is the main stimulus that the teacher offers, and certainly not twos and fives, which of course stimulates something, but, unfortunately, not what is required.

Deliberateness means that as much time is given to each difficulty as needed. It doesn't matter if not many topics were "covered." That only matters when something must be "covered" by a certain deadline and regardless of whether it is covered well or poorly. If that is all that matters, since in the end nothing has been covered, then everyone—both pupils and teachers—lose interest.

Independence means that pupils independently perform a substantial portion of the theoretical material and sometimes nearly all of it: they prove or disprove the majority of problems and theorems on their own. A direct description by the teacher is ineffective (Konstantinov, 2001).

Prevalence of Free-choice Informal Voluntary over Formal Forced Education

Studying museum education, John Falk, Lynn Dierking, and their colleagues have come to the notion of "the free-choice learning environment" (Falk and Dierking, 2002; Falk, Donovan, and Woods, 2001). In the free-choice learning environment, students (e.g., museum visitors) have a choice to come or not to come to the educational institution (e.g., museum), to come to any display they want, and to engage (or not engage) in the exhibit in a way they want, to define their time, people, communication, movements, and so on. In contrast, in conventional schools, the learning environment is assigned and forced on the students: everything—education, time, space, people, communication, curriculum, instruction, relationship, activities, and so on—has been predetermined and decided instead of and for the students (Matusov, 2015a). Konstantinov came to the idea of the free-choice learning environment through observation of the "successes" of forced education: how it distorts the students' initial intrinsic motivations and passions for math:

> [Slava Tsutskov] led a circle for schoolchildren in quantum mechanics [in the mechanics and math department of the Moscow State University, MGU]. But he couldn't describe any mathematical tool to them because they didn't know anything. So then we decided to make a circle for providing the math tool that they could use to write and understand all the equations of electrodynamics and quantum mechanics. That was the Beta circle. And the people from the Alpha circle were assistants. Since there was a firm objective that they master something, we had to impose a fascist regime. The discipline was like in the Gestapo. Anyone who failed to complete even one assignment would be expelled from the circle. But he could come if he completed the assignment. Seventy people made it to the end. (Dorichenko, 2010).

NK: About 200 people came to the first class. It made a completely astonishing impression. We sat in a big auditorium, there were two chalkboards, on opposite sides. Vitia Pan conducted the class on one board, and I on the other. People who sat in the middle could look at either side. Later the circle stabilized, and 70 people were left. Slava Tsutskov and I had agreed that he and I would teach a course in [math] analysis and would go as far as Maxwell's equations and Schrödinger's equation. But in order to actually cover all this, people could not miss any sections. So I announced that anyone who failed to complete an assignment would be dropped from further classes in the circle. This was a very stringent requirement. The result was that all 70 people completed all the assignments. But as soon as I dropped that stringent requirement, almost everyone stopped doing the assignments, and then stopped coming to the circle altogether. They came for as long as

> they were afraid of being kicked out. I often cite this example to show how stringency leads to a distortion of objectives. Now I tell my pupils: "If you are hoping I will make you work, don't hold your breath!" Because it's true, it is possible by being tough to make sure they learn something. So what? Is that really the highest objective? They will learn something, and will know it. So what? There won't be any point unless there is an inner stimulus to act on it (Konstantinov et al., 2002, p. 45).

According to Konstantinov, genuine education requires freedom of choice for the students' interests. When asked if he had all levers of power, how he would change education, Konstantinov replied that he would stop the mandatory military draft for male students in Russia. Many of male Russian students want to go to college to be excused from the draft. Without the military draft students would be "free to choose their interests" (Privalov, 2012).

In my observation, Konstantinov's free-choice learning environment was based on several practices:

1. Students' voluntary attendance and participation in diverse math Olympics and math competitions at many levels;
2. Students' voluntary attendance and participation in math circles;
3. Students' voluntary choice of selection of math problems-theorems on leaflets in math circles and math classes;
4. Students' choice of engaging or not engaging in solving math problems in the math analysis classes;
5. Students' organization of their own time;
6. Students' choice of movement, association (or not association), collaboration (or not collaboration), listening (or not listening) during the math analysis classes;
7. Math analysis teachers' full or partial sabotage of summative assessment (grading);
8. Konstantinov's interest in and support of students' diverse non-math interests and talents;
9. Availability of rich choices of extracurricular voluntary activities for students (e.g., kinofak, literature facultative, hiking, KSP—authorial signing in woods, summer camps, and so on).

EM: Sometimes we would come to a dead end and then we would ask Venia or Andrei or another undergraduate to explain—and they would do these mini-lectures for anyone who was interested. If someone wasn't interested, they would sit further away and do their own thing. And if memory serves, we would sometimes leave and work on these problems outside the classroom. We would even go with friends to a small yard in front of the school and we would work there until the front office caught us, and we would say that we were allowed to leave. And despite the fact that we were able to work or not work on these problems in the course of a week, as far as I recall, I had enormous experience in solving those problems. And no one hovered over us and made us do a certain amount. At least I don't remember being pressured, for example, to do three problems by Friday.

NK: You are right about all of that. Now we have some people who have gotten to be teachers who cannot understand this. They impose very stringent requirements, that such-and-such problems must absolutely be handed in by such-and-such a date, otherwise they will propose expulsion from the school. We cannot get on the same page with them because these people themselves never studied. For example, one attended a pedagogical institute and then dropped out. Clearly, this is not taught at a pedagogical institute.[31]

EM: I think they teach the direct opposite at the pedagogical institute.

NK: Absolutely correct. They teach the direct opposite. And I know that at one school, which is called "The Intellectual," one person who cares about everything, he is a physics teacher, says: "When someone comes and says he also wants to work at our school, I always ask whether he came from a pedagogical institute. If he came from a pedagogical institute, I try to get rid of him right away, because he holds completely opposite ideas." So what you said about the pedagogical institute is correct.

EM: Although I myself work not at a pedagogical institute but in a pedagogical department, I do try to inject something that is not traditionally taught in such departments.

NK: Instructors who study my experience are much more liberal about how pupils learn. For example, in a strong math class, where all the pupils but one are winners of Olympiads and one pupil has not gone to a single Olympiad. But he is good with his hands. And so the instructor, a strong mathematician, forgave him for spending almost no time on math. It did not occur to the instructor to drop him because he did not do what the others did. But he is so good at working with his hands that when they now started to introduce the subject of technology instead of shop (the subject of shop was eliminated), where you need to make some device, for example. First you use the computer to make a model of the instrument, and then you make it in metal. In other words, this is completely different from what the subject of shop used to be, where you had to work with a hammer and file. Anyway, this fellow did everything very well both on the computer and in metal, but he did not solve math problems. Anyway, he was so successful in that work that the rector of the Bauman Institute called the school to find out whether this fellow could enroll in his Bauman Institute. This is the only time I know of when the rector himself has called and asked whether a pupil has enough knowledge to enroll in his institute (Conversation between EM and NK, part 1, October 30, 2016).

The three major elements of Konstantinov's math education—math competitions, math circles, and math classes/schools/university—work together to organize the free-choice learning environment:

Russia has developed a comprehensive system of math education for older schoolchildren. The word "system" may not quite fit. Many parts of the system are not that closely connected to one another and are not harmonized enough in content and style to speak of a "system." These parts have been created through the efforts of tens and hundreds of higher educational institutions, math schools, various regional collectives, and individual activists. Nevertheless, the commonality and cohesion are still apparent. The principal parts of the system are:

1. Math schools and classes.
2. Municipal math circles.
3. Summer math schools.
4. Math-based correspondence schools (AMCS, PTCS [Physics and Technology Correspondence School], the school of the Small Mechanics and Math Department, and some others).
5. The journal *Kvant* and other publications to assist teachers and advanced learning schoolchildren.
6. The Russian Math Olympiad with all its stages, from school-oriented to the All-Russia Olympiad.
7. Other republic-level Olympiads (the Soros Olympiad, the International Tournament of Towns, about 120 towns in 25 countries, with more than 10,000 participants in 1999 – 2000).
8. Regional competitions (the Kolmogorov Cup and tens more competitions).

As a result, two groups of activities stand out: those aimed at education (items 1 – 5) and competitions (items 6 – 8). The core of the system is education, while competitions serve to fill up the classes and clubs, and in addition, they are an ornament for the system that gives the education an aura of a big holiday. I will call the entire system "Russian math classes," underscoring the leading role of education and without forgetting that this name is somewhat one-sided and does not reflect the entire abundance of options for working with schoolchildren (Konstantinov, 2001).

Authorial Pedagogy

Konstantinov's pedagogy is authorial. This means several things. First of all, a Konstantinov's teacher is expected to be a person with wings actively involved in self-realization, self-actualization, self-inspiration, creativity, and passion in math, pedagogy, interaction his/her with students, and probably some other spheres.

NK: I got into this work partly prepared: there were already leaflets, “sheets of paper” [with math problems]; there was a curricular program, which of course varied. There are as many versions of curricular programs as there are math classes; they are always **authorial** (Borusiak, 2010).

Second, it involves innovations and experimentation in response to changing interests of the teacher, encounter with students' interests and challenges, and addressing emerging challenges.

NK: A small correction. I did indeed come up with a lot of ideas, but a large portion of my attempts failed. It only seems like everything worked out. In fact there were many more trials. Maybe there wasn't enough energy and enthusiasm. But some things did work out, for example, the Lomonosov Tournament and the Tournament of Towns (Borusiak, 2010).

Third, it involves ownership of the curriculum: the math analysis teachers' right to develop their own curriculum topics, leaflets, and math problems. Thus, for example, with a help of teaching partner, Venia Dardyk designed a new curriculum of Markov's replacements based on his own math interests at time. Alexander Shen reported, “all the leaflets were written ‘from scratch,’ without any copying and pasting, but different sources for problems were reviewed, of course. Actually, one can compare the leaflets from 2000 and 2004 to understand how similarly the same topic was presented. . .” (email communication, December 4, 2016).

EM: That's interesting. Now, you mentioned in an interview with Liudmila Borusiuk that these were authorial programs. In what elements do you see this authorship? For example we had Venia Dardyk and Andrei Pechkovskii.

NK: They were within the range of the overall tradition. What elements was the authorship apparent in? First, they thought up problems themselves for the scheduled classes. . . All the problems were thought up. But some were repeated—you can't get by without certain standard facts. There were very big changes. Some topic could be transferred to another grade. From the 8th to the 9th or vice versa. It was in that respect that they were authorial.

EM: And I thought that all those problem leaflets were standard.

NK: To some extent they were. There were two or three cases in which those leaflets were published as a separate little book. School. No. 57 published such leaflets. School No. 179 did, too. Why were they authorial? I will tell you why. A great deal of work was done by the teachers. Pechkovskii and Dardyk were the authors, while other [college] students were assistants. One of the main jobs that a teacher does is to think up problems and make [pedagogical] decisions. Suppose there is some math class or a class that is contending for that name. The first year we thought up problems, and for the following year we used the problems that had already been thought up. But then we thought that that was worse. It is better when the people teaching math are the authors and when real situations always crop up in class. Rather than what they know in advance and taking problems from some textbook.

EM: But why is that the case, could you explain that? This is a very interesting point. Why is it so important to think up problems all the time? Why can't you use what was thought up already a few years ago and continue to use it?

NK: In part, of course, you can.

EM: I understand. But why is it so important to update all the time? And based on what? Based on what new kids come in or something else?

NK: I don't know how to describe it. It's just that when I am thinking of what problems to give a class, I proceed of course on the basis of how they are currently solving problems. I can see, after all, what is working for them, and what isn't. I think up new problems. Sometimes very simple ones, incredibly simple ones, that turn out to be important.

EM: You know, I am going to share a little bit with you again. With age I have begun to forget some things, and I have discovered that this is a huge plus, because I stop repeating myself and always invent something new. Because I already forgot how I did it. Because when I invent something new, I acquire those "wings" that you talked about. But if I try to recall something that I invented in the past, I don't have those wings. And I don't inspire my undergraduates because I am trying to recall instead of having this upsurge. And sometimes I am surprised and wonder why it was so good last time and I try to do the same thing, nothing works, because I myself no longer have the enthusiasm. But when I don't have it, my students don't have it, either.

NK: Yes.

Fourth, it is the teacher's interest in listening to the students' solution for the students' own mathematical voice. The student's mathematical voice is defined by the student's mathematical authorship—his /her original creative ideas, his /her unique personal meanings, his/her personal math passion, and his/her math aesthetics—however common and agreeable the math outcome is.

NK: First, we have this phenomenon here. Arnold, the famous mathematician. … At a seminar given by Kolmogorov, Arnold described his solution to a version of Hilbert's 13th problem.[32] There is a somewhat tedious proof and a lengthy one. And at one point he began speaking very loudly for some reason. And Vitushkin, who also participated in this proceeding, asked me: "Have you noticed that he described that point very loudly for some reason?" "Because it's the only point that he thought up himself." In other words, the rest of it was ordinary, and he needed to think up something surprising. And that made him enthusiastic, but when he was doing something trite, he was not enthusiastic about it at all. I would put it this way: when a person has solved a problem himself, you can figure out, based on how he describes it, where the most important point is, where he overcame some internal barrier. But if it's not the main point, it's uninteresting.

EM: In other words, Dardyk and Pechkovskii tried to listen for those most interesting spots, where the most interesting things occurred. What else? When you work with a pupil who is describing a problem to you, what else is important?

NK: I think so.

Fifth, the teacher's pedagogical authorship involves recognition of difficulties, challenges, and falsity in which the students may be involved but not yet recognize for themselves. Revealing these challenges and difficulties is important for promoting the student's mathematical voice.

NK: First, you have to pick the right time to carp. When someone screws something up, you have to look at why he screwed it up. Because it is stale and uninteresting for him or it is unclear to him and that is why he is making the mistake. You want there to be total clarity. I had a pupil who put it very well. It was at a session of a circle; classes were over already, and the circle was continuing to work. And some women from television came and asked, "Why are these kids continuing to work, even though classes are over?" We said it was a math circle. They were surprised that a math school also has a math circle. And they asked with astonishment: "So you probably love math?" And one of the circle members replied: "It's the only science that does not accept lies." That is, it does not accept them at all, ever. And after that I began to understand that math and truth are almost synonyms. Now, in chemistry you can fib, and in physics you can fib. Although physics also dislikes lies. But still, not to the same degree. Not to mention the other sciences.

Sixth, it is finding and revealing the unexpected math beauty in the students' solutions guided by the teacher's own mathematical interest and aesthetics.

EM: That's interesting. Now I recall the work of our instructors. When they would check our problems, one of their duties was to try to find a hole in our reasoning. But I also remember that Andrei liked to find aesthetics in the solutions to problems. So I had a dual sensation as a pupil. On the one hand I had to protect myself, since they were looking for a hole in my proofs. On the other hand, this inspired me very much, because Andrei liked to find beauty in this. Sometimes I also understood that it was beautiful, but sometimes I couldn't understand why it was beautiful. Where did he see this beauty? But it was very helpful. Because if he had only focused on finding holes, I'm not very sure that it would have completely inspired me.

NK: I can add to that now. We have an instructor named Slobodnik; he himself studied in our very first math class. He's a very good mathematician. Anyway, even in the most ordinary proofs, even in a textbook, he can find such a surprising twist that it becomes simpler and more beautiful. He puts great value in it when he manages to find this. And he also hears out the proofs of his pupils and helps them to find beauty in this.

EM: I also liked this—that close listening. I remember both Andrei and Venia would attentively listen to what we were describing, without interrupting, especially at the points that were important, and tried to understand our thinking process rather than propose some ideas of their own, which might not have been as important to us. I remember that what impressed me about them was that this was in stark contrast to my regular teachers, who immediately said how it should be done. . . .

Additionally, when this listening process is going on: when you listen to your pupils, listen to their solution to a problem, what do you focus on?

NK: Well, I don't pay attention to some things: if it's stale, and there's nothing interesting there. I may say flatly, you don't have to prove that, you can move on. Clearly, if you pay attention to every little detail, that will become a pain in the neck.

EM: In short, you proceed according to your interest? Your interest determines it?

NK: Of course. You remember, I told you about Arnold? When someone thinks up something himself, accentuates that. You understand?

EM: In other words, you also listen to the pupil's intonation? That is, it's your interest plus the pupil's intonation, which focuses you on what is important.

NK: Yes. (Conversation between EM and NK, part 2, November 2, 2016)

Although Konstantinov had not come to the notion of authorial learning, he accepted it in our conversations. In his view, students' authorial learning emerges in a response to the teacher's authorial creative teaching.

EM: Now I would like to ask you—we spoke about authorial teaching. Do you think there is such a thing as authorial learning? That is, when a pupil learns, it is, in a sense, authorial learning. Two pupils learn in absolutely different ways.

NK: I wasn't thinking about that. But in certain cases that is obvious. Here is what Kronrod reported. When he was a schoolboy in the lower grades, his father was exiled. He was exiled to do logging work. At the logging camp in the Komi Republic, in addition to our own people, there were also German engineers, whom the government had invited. The German engineers had prepared in a serious way and not only came and lived in the forest where the logging was taking place; they had also brought a German teacher with them. And since there were no other teachers there, Kronrod learned in German in the lower grades. What was distinctive about that was that pupils in the same class ranged widely in age and were on vastly different levels. That is, the teacher had to simultaneously teach math to all of them. And one assignment was like this (for all pupils, regardless of age): you had to cut a triangle, any kind, out of a sheet of paper. So all of them did the homework assignment and came in. Naturally, all the triangles were different. He told everyone to do this assignment: cut all three angles out of the triangle. Then lay them next to one another. And these angles formed a single straight line. He even asked them: "So did it turn out for everyone that they formed a single straight line? You probably arranged it with one another? No, you didn't." In this way, he instilled the idea in them that the sum of the angles of a triangle equals the sum of two right angles, and did so in such a form that they were very surprised at that fact.

EM: Very elegant! Wonderful!

NK: I think these are elements of authorial pedagogy. When someone has learned how to surprise others with something that is astonishingly primitive and simple, that's as good as it gets. In fact, it is very profound. Of course, a teacher should know how to find such methods. And it is very easy to make all math primitive, simple, and uninteresting if all the facts are so obvious that there is nothing to think about.

EM: As a pupil, I was always interested in why people would think this up, why this was important. For example, why is it important that the sum of the angles of a triangle equals 180 degrees? And why is it important to provide such definitions to a continuous function? And until I was able to understand this in each concrete example, I was unable to move forward as a pupil. Until I understood the depth of this question, and not the answer and not the quest for the answer, but why people ask the question. Who came up with the idea of asking that question? What is that important question for? As soon as I answered that question internally, I could already do a lot. But as long as I was unable to answer it, it was a stumbling block for me (Conversation between EM and NK, part 2, November 2, 2016).

EM: You know, I kept thinking after our conversation yesterday about your example, you were describing the teacher who asked pupils to cut out a triangle and then cut off the angles and lay them down next to one another. I kept thinking, why do I like that so much? And here is what I came up with. There are so many interesting layers in this approach. One interesting layer for me is the relationship between geometry and algebra. When the sum of the angles is asked, that is, a measuring problem, you need to measure the angles and add them up. That is working with numbers. But he turned this into a geometrical problem, because adding up angles means taking the angles, cutting them off and placing them together and seeing what the result is. There is another interesting layer —the pedagogical one. After all, what did he do—when the children did that, a slew of interesting questions arose —for example, why is that the case? Is it always the case? Maybe with some other triangle it will no longer be the case. In other words, it's a provocation of sorts. I recalled that many things we had in school and in the circles were based on such provocations, which came up as a result of something surprising like that. And there were these two common questions—why? And will this always be true? And for me these are pedagogical questions. The third layer: it's interesting that he incorporated something from physics into mathematics. Because in physics we like to experiment, what will happen if we do this and this. And a certain pattern emerges, I don't know what that is in Russian; in German, it is Gestalt. In this case, everything lies on a straight line. The angles are placed alongside one another and the result is a straight line. This straight line is a type of Gestalt or pattern. But it is unclear why it emerged and whether it will always be like that.

NK: Yes, of course, interesting questions come up. I think many teachers don't notice how many interesting questions they miss.

EM: Absolutely right. And it seems to me that all this came up in the math circles. Maybe no one purposely thought this up, the way I am saying now, but you could sense it, and these questions were there. And this connection with physics and math is very interesting, because from the standpoint of physics, 30 experiments were conducted if there are 30 pupils in the class, and we see that the experiment was successful 30 times. But from the standpoint of mathematics, this is not a major proof.

NK: Well, in part, it is still a proof when it comes out once and then again, and then you have to start thinking, is there some theorem here?

EM: Yes, from the math standpoint there could be some theorem behind this, and from the physics standpoint there is some natural law behind this. … Somewhere I read that you said, in some interview with you, where you were discussing something similar, when one teacher, in math, was putting pages together—there was some perpendicular line, and the teacher was displaying this by placing pages next to one another (Conversation between EM and NK, part 3, November 3, 2016).

Authorial teaching and authorial learning (Matusov, 2011) requires freedoms recognized and promoted by the institutions and even by the entire society. Arguably, this recognition is currently missing from all modern societies. In the following section, I will discuss objections to Konstantinov's pedagogy that in part emerge from a lack of such recognition.

Part III: Objections to Konstantinov's Pedagogy

From my observations and interviews, I have abstracted three major objections and challenges to Konstantinov's pedagogy:

1. *Social justice*: Math schools generate the brain-drain from regular public schools.

2. *Tension between students' dilettantism and professionalism*: Students' capricious choices of math problems/theorems may handicap the emergence of the students' necessary knowledge and skills for success in the math practice.
3. *Lack of support for math analysis teachers*: Giving freedoms to math analysis teachers may leave them to their own devices both philosophically, pedagogically, and mathematically.

In the following sections, I describe and consider these challenges.

The Brain-drain Burden on Regular Public Schools

Social Justice Objection against Math Schools

In 1971 – 1972, there was a debate regarding math schools between two famous and powerful Soviet academicians: physicist Pyotr Leonidovich Kapitsa[33] and mathematician Andrey Nikolaevich Kolmogorov[34] in the politically and ideologically powerful Soviet journal "Issues of philosophy." Kapitsa argued specialized schools were harmful for overall school education because they involved a brain drain of advanced, highly motivated students, leaving general Soviet schools with unmotivated and weak students. This brain drain weakened peer guidance and peer support, which could be even more important than teacher's guidance, according to Kapitsa, and deteriorated the overall intellectual academic environment and potential of general Soviet schools. Even more, Kapitsa argued, that a lack of weak students in specialized schools, such as Konstantinov's ones, harmed the advanced students because they did not have opportunities to provide peer guidance and learn from it. In his response, Kolmogorov reminded Kapitsa with an irony that both of them were a product of specialized schools in pre-revolutionary Russia and that there was clear evidence that many alumni of specialized schools highly contributed to the Soviet sciences. Additionally, Kolmogorov pointed out that Kapitsa's objections were speculative and not based on any evidence. In his reply to Kolmogorov's rebuke, Kapitsa clarified his position that he was not against specialized schools as such, but he was concerned that their innovative pedagogical practices were not spread to general Soviet schools (Denisenko, 2017).

NK: There is also opposition to math schools. When strong pupils are plucked up for math school, a teacher is left with weak pupils. The class loses a leader. It may be better for the pupil, but it is worse for the entire class. So, for example, Pyotr Leonidovich Kapitsa was against math schools. He felt that one strong pupil in a class changes the situation a lot (Leenson, 2012).

NK: Germany, for example, has no math schools. The feeling is that it is a breach of democracy and all schools should be identical. If a pupil is talented, he gets help; for example, he is given an opportunity to get books from a university library. But the necessary environment is not created for him. That is wrong: a person should have a sense of life in another environment, in another world. So undergraduates come to our school endlessly and voluntarily take on various jobs. Why do they come? My hypothesis is that the environment here is good (Leenson, 2012).

Konstantinov's and Matusov's Replies to Social Justice Objection

NK: I would respond this way [to Kapitsa's objection]. When a pupil studies in a milieu of average pupils, there is little he can learn from them. This really hit home with me when I graduated from school and enrolled at the university. There were already undergraduates there on a level that I did not have in school. And now one young woman, who is 40 years old, which to me is young, said this. "What is a good university?" "It may just seem that a good university is one that has good teachers." "But I feel that it is very important who a person interacts with when he enrolls in the university—that is even more important than what kind of instructors there are." So I thought to myself that that is correct. A person gets a new milieu for interaction, which is much higher than in school.

EM: I liked Kapitsa's criticism because it is thought-provoking. He was, of course, a very smart person. I understand that Pyotr Kapitsa was concerned about what was happening to everyone else. In other words, it is clear that you are creating a good milieu for those who "sprout wings." But what do you do with the others? It seems to me that if you help young children, all of them sprout wings. Then a conventional school, and not only school, clips those wings. Later, for some reason, some kids still have those wings. But if the wings are not clipped from the very outset, then in math and other fields of activity one can create centers that support those wings. And it seems to me that if we try, although it may be utopian, every child will get into either one or another, or several at once. In other words, the problem that Kapitsa refers to, he says this because there are institutions that clip wings. But he wants, in a sense, to destroy the solution to this situation. To my mind, his suggestion is very dangerous—in a way, if we cannot make everyone happy, then let us make everyone unhappy [a famous Soviet joke]. So I am on your side, not on his side, despite the fact that I understand his concern about what to do with the others, unmotivated and weak students. My answer is that we have to come up with something for the others to promote their wings as well. Not only math schools, but many other specialized circles and classes where those wings will be supported.

NK: Here is what I would say in this regard. Right now a kind of concentration—many talented people—has formed around our school No. 179, School No. 57, and School No. 239 in St. Petersburg. And when I took a look, all of them are strong Olympiad participants and jury members of the Tournament of Towns—it's a very strong group, math-school instructors. And if you take this whole strong group and try to compare it with another group—

the School of Athens, when Athens had all those mathematicians, scientists, and philosophers—I think the strong group that we have formed could be even stronger potentially than the School of Athens. The School of Athens includes all those great mathematicians. Maybe, I don't know of course, maybe it will create something in science. Something so new that it will surpass the achievements of the School of Athens. Maybe. So you get my thinking? That is what I think in this regard. Maybe a completely new stage in science will emerge thanks to this concentration of strong people. That is my hypothesis, that maybe such a concentration of strong people could lead to extraordinary results in science.

EM: It seems to me that they are occurring as it is. You have probably heard of this person [Edward] Frenkel (Frenkel, 2013b)? He is a math professor in the U.S., at the university in Berkeley. He is a superb mathematician, a math-school graduate. And if you look how many superb scientists have graduated from math schools, it is a tremendous contribution to science.

NK: Yes, that is precisely what I am talking about. But it is very important not to ruin it. This opportunity can easily be ruined by issuing laws that make such advancement impossible. (Conversation between EM and NK, part 2, November 2, 2016).

Tension between Closed and Open Curriculum

One of the biggest tensions in Konstantinov's authorial pedagogy is a tension between closed and open curriculum, in my view. The closed curriculum involves the teacher unilaterally deciding what the students must study, in what order, and through what learning activities. Closed curriculum pedagogical regime tries to ensure that all students will go through learning experiences that are considered important from the teacher's professional judgment. Closed curriculum is aimed at providing holistic well-rounded education in an academic subject. It can also try to ensure that a student is ready for a new curricular challenge while using the previous learning experience as a tool and springboard for the new curricular challenges. Thus, a previously solved math problem-theorem can become a tool for solving a new math problem-theorem. Without having solved important math problem-theorems, a new math problem-theorem can become overwhelming for a student.

However, since all these decisions are unilateral, closed curriculum unavoidably involves non-negotiable impositions on the students (Matusov, 2015b) and even pedagogical violence, in case a student resists or is not compliant with the impositions (Matusov and Sullivan, 2017). Closed curriculum is based on forced learning. The imposed, forced nature of closed curriculum often distracts the students from their interest and ownership for their own education, leading to pleasing the teacher and doing busy work of the imposed assignments (Blum, 2016). Konstantinov was faced with this dilemma of closed curriculum early on, while he was teaching math circles, which forced him to shift to somewhat open curriculum:

EM: You know what I want to ask you—I read in an interview with you, when your circle was still in the MekhMat department [of MSU]—regarding, I think, Schrödinger's equation. You said that if anyone doesn't do an assignment or doesn't come in, you will kick him out. And all 70 people, as one, did all the assignment. But then later, when there was no threat of expulsion, many stopped doing the assignment and stopped attending. And the circle died.

NK: Yes, that happened. You are correct.

EM: You came to the conclusion, the paradoxical conclusion, that you can't make pupils work all the time, that some balance is needed. Could you talk a little bit about that experimentation? I think you were experimenting for a while; do you remember what took place?

NK: It was the Alpha circle. Indeed, I wanted the kids to learn some basics of math analysis, and this required that absolutely all of the problems be solved. You can't get an education piecemeal. There has to be some holistic quality to the education. So I forced them and said that if anyone didn't do an assignment, I would kick them out of the circle. But later I realized that that was over the top, and I switched to a very liberal system.

EM: I would like to ask you this. It is clear from your experience that there are problems on both sides. In other words, if you force them all the time, then people's values change. The value switches from studying math, from being inspired, from having wings, to not being kicked out, or to getting a candy, or something like that. But on the other hand, if you only use a dilettantish approach… by the way, the word "dilettantism" is not a bad word, it derives from an Italian word that means the same as the English word "delicious." And that is great if it is "delicious," but the problem is that things are left unexamined, sometimes there isn't enough experience. Because when you focus only on what is delicious, you can miss out on something that is very important. It may be "bitter," but important. What kind of balance should there be? What do you think right now? How do you strike this balance?

NK: Of course there has to be a balance.

EM: I had a graduate student who was working on a dissertation about this, not in math but another field (Anderson, 2010). He was also working with children, and in the dissertation, he talked about dilettantism versus professionalism. There is an interesting tension between them. He argues that one must definitely start with dilettantism, and later, at some point, switch to professionalism. He has that interesting point of view. An interesting pedagogical tension arises here. He argues that rather than a balance there should be a transition from one to the other.

NK: I never thought about that and I don't know (Conversation between EM and NK, part 3, November 3, 2016).

Open curriculum pedagogical regime involves a free-choice learning environment, one in which each student decides for him/herself whether to investigate a learning activity and what order in which to investigate that curricular learning activity. The teacher's responsibility is to provide choices of curricular learning activities and guidance. In open curriculum, the guidance mostly involves helping the students in the learning activities of their choices: helping the students when they get stuck, testing the students' solutions, replying to their questions, and so on. Additionally, this guidance may involve helping the students manage their challenges. Remember my teacher Venia Dardyk advised me to stay longer on my challenge of understanding of the continuous function definition. The teacher can also direct a student to another math problem that may be necessary for solving the problem that the student chose.

Open curriculum promotes students' wings by engaging the students in decision making and ownership of their own education. When a student is involved in solving a problem of his or her choice, with high probability this activity is driven by the student's math interest and math desire and not by the teacher's imposition. Open curriculum promotes voluntary and interested education, which is the basis of wings: students' self-inspiration, setting themselves on a (learning) journey, self-realization, and so on.

At the same time, open curriculum may create curricular holes in a student's knowledge and skills, and a fake (if not even arrogant) sense of expertise in the student, as result of the student's emerging dilettantism of doing only what the student likes to do.

Interviewer: There seems to have also been criticism of math schools over the fact that pupils there have to be forced to do what in a math circle is done out of sheer interest.

NK: That did happen. There was very sharp criticism of math schools. The point was that circles rested on the idea of being a counterpoint to the school. And for the schoolchildren who were going to circles, a devil-may-care attitude was cultivated among them toward twos [failing grades].

Interviewer: But didn't that attitude largely transfer to math schools as well?

NK: In part, it did. But the whole point is that once you move to school, you can no longer cultivate the idea of hating school [in the students]. Instead, our authority exalted all the shortcomings of [conventional] school. That [i.e., losing the opposition to conventional schooling as a positive force for the authentic math education] was the main objection to math schools [at the time]. Plus the fact that [in conventional school] pupils would be under pressure to learn what they had previously learned voluntarily [in informal math circles]. That is correct.

But on the other hand, you cannot get by in math only with the fact that you feel good right now. You need to overcome challenges. You cannot learn anything serious if there is nothing to overcome anywhere. There was actually a great deal of superficiality in the circles. We would hear there "Oh, those are Diophantine equations![35] We covered those, we know it all!" "What do you know? Do you know how to solve one?" "Uh, no, let me think a minute…, it gets solved somehow" (Konstantinov et al., 2002, p. 47).

Open curriculum also may create a sense of unused and missed learning opportunities. Thus, one of my classmates from the School No. 91 told me in our conversation that he wished our math school had had a strict order and "oppressive grading" (his words). He felt that because of too much freedoms and mess in our school he had missed a lot of educational opportunities in our school, important from his current vista. He told me that as a person he must be forced to engage in important experiences. He remembers that his most important and valuable experiences were always forced on him. He stated that those students did well in our school who wanted to learn; the others felt through the cracks. However, at the same time, he admitted that he learned math deeply under our open curriculum and free-choice learning environment in our school—the fact that apparently surprised him in our conversation. Thus, according to his account, he learned math well in Konstantinov's school but apparently he did not develop wings of self-realization, self-actualization, and self-inspiration that might be cut for him before he came to the Math School No. 91. I doubt that the strict order, "oppressive grading," and forced education would have promoted wings in him but it raises a question of what kind of educational-therapeutic environment might have helped him and students like him reconnect with his own authorial agency (cf. Matusov and Marjanovic-Shane, 2017).

Also, this emerging dilettantism, inherent in open curriculum, can become overwhelming for some students because the students may inspire to solve math problems, which they are not ready to solve yet. These students may become so discouraged that they lose their wings so to speak by stop engaging in the curricular activity at all:

NK: Yes, the problems there [at the math circles] were given in quite an unsystematic way. One person solved something, I went up to him and asked what the solution was. Then, someone else solved it; I went up to him and asked what the solution was. There was no system there. They were simply different, random problems. Additionally, there was no sequencing at all; first, you need to solve this one, and then this one. They were all simply a test of acumen. And therefore this transferred to the classrooms as well.

EM: I think this is extremely important—that the problems were chosen by the pupils themselves, that this selection was part of the learning process. Because this is how a proactive educational attitude is manifested, now that is the authorship of learning. I think this is a very important point, because in conventional schools this authorship is missing. And if authorship is missing, that winged quality diminishes. And if you take away that freedom of choice, it doesn't matter whether the choice is made according to a scientific system or not according to one, there will be no choice anymore.

NK: No, there is a system all the same. Indeed, when we give out some problem, we see whether people have enough knowledge to solve it. In other words, we don't require them to use knowledge that they don't have yet.

EM: That is clear. It's like during the selection process for the school. There were problems that could be solved. Of course, Hilbert's problems were not given to the 7th-graders. Although, on the other hand, if some 7th-graders obtained Hilbert's problems and decided to work on those problems, that would not be an issue, either.

NK: But there are people who are more or less typical—I told you about one guy I had who didn't solve any problems—he only learned the Spanish language better than anyone in Moscow, and I didn't kick him out, although I could have.

EM: I apologize for interrupting you, but in that example he was the one who decided what was important to him. He decided that math did not matter to him so much for some reason.

NK: No, that's not quite true. There was one problem in math that he liked very much. And he spent a long time trying to solve it, but never did. He went past the solution several times, he didn't realize that he had already solved it, and thought that he hadn't. And everything would have gone well for him if he had solved twenty easy problems, but he didn't do that, so it was hard for him to what is usually not hard for people. In short, he is a good guy, but he isn't a mathematician.

EM: And if you had offered him those easy problems, would he have refused or agreed to try to solve them?

NK: No, I offered them to him, of course, but he was occupied with other business. After all, he did take first place in Moscow among all the pupils in Spanish-language schools. And taking first place in anything is always difficult. I understand, after all, how difficult it is to be

> first in something. And that's it (Conversations between EM and NNK, part 2, November 2, 2016).

In my view, both closed and open curriculum pedagogical regimes create "curricular holes" and "unreachable curricular abysses" in students' knowledge and skills but for different reasons. In closed curriculum, although the teacher tries force each student to "march through" the entire holistic field of the curriculum, the student mind and heart remains selective. In certain curricular learning activities, the student may choose to actively engage and in certain not to engage. Also, relevance of the curriculum for the student stays selective. This selectivity in the student often remains invisible for the student and the teacher (and peers) and thus stays unguided by both of them. Additionally, since all students march through the holistic curriculum in a locked step, the students' learning is unavoidably, partially, fragmentally, and insensitively guided. Thus, using Lave's (1992, April) terminology, although "the teaching curriculum" (i.e., what the teacher tries to teach) is holistic in closed curriculum, "the learning curriculum" (i.e., what the student actually learns) often remains fragmented and porous; the phenomenon that is increased with time after the instruction.

In open curriculum, the student's learning curriculum can be fragmented and porous due to the selective nature of the student's learning activism and dilettantism—interests, attractions, and desires—or lack of it for particular curriculum learning activities, theme, and even subjects. However, I argue that students have at least two advantages in the open curriculum pedagogical regime over the closed curriculum pedagogical regime. First, although both regimes may be faced with "curricular holes and abysses," students in the open curriculum learn how to be autodidact—that is, they learn how to learn—and, thus, can organize and manage their learning when they are faced with curricular holes in their knowledge and skills. Second, students in open curriculum have wings that promote their learning interests and activism to *desire* to learn. In contrast, students in closed curriculum often lose their wings due to the fact that learning is forced and imposed on them and the lack of ownership of their own education. I suggest that we should expect and normalize curricular holes and abysses and prepare our students for dealing with them.

Finally, Konstantinov brought another relevant problem: human wings are selective and not universal. Not all people are interested in math and this should be expected, normalized, and not fought. In the natural world, well-rounded, non-specialized, organisms are doomed to mediocracy and would fail evolutionary competitions with specialized organisms. The same seems to be true with subjectivity of the human mind. When all subjects are special for a person, nothing is special. Human wings—self-realization, self-actualization, self-inspiration—are selective. Educators should expect that some academic subjects and activities will be central for the students and some peripheral. Some students will become inspired by math, some by Spanish, some by handcraft, and so on. This requires developing diverse learning communities of curricular subjects with central and peripheral dynamic membership. Konstantinov seems to envision these diverse learning communities by organizing schools with diverse curriculum foci: math community classes, biology community classes, literature community classes, and so on. He has also organized diverse informal circles and diverse curricular Olympics.

Observing Konstantinov's pedagogical practice and conversing with him about that, I have come to a conclusion that although Konstantinov does not fully reject closed curriculum, he gravitates toward open curriculum. When recently was asked about how much freedom should be given to a student, Konstantinov replied, "It depends on a student. For example, when I was a student, I required as much freedom as possible. However, it does not mean that this freedom should be given to all students. However, it is true that without freedom a student cannot study."[36]

Tension between Pedagogical Freedom and a Lack of Support in Authorial Pedagogy

Apparently, there has been very different support and guidance that Konstantinov provided to first time teachers of math analysis in his math school, who were college students at the time and who had been alumni of Konstantinov's math schools. Some of the math teachers were in close and systematic contact with Konstantinov (like Venia Dardyk and Andrei Pechkovskii, 1974 – 1977) but some were completely left to their own devices (like Alexei Riabinin and Oleg Kazakov, 1979 – 1982) and some in-between (like Alexander Shen and Sergey Dorichenko, 1980 – 1983). Oleg Kazakov reported me that he and Alexei ran their class more like Konstantinov's math circles by mostly using leaflets from their own class when they were students. It was more reproducing the previous pedagogy that they had experienced themselves as students without much pedagogical authorial activism and enthusiasm. I interviewed my classmate Alexei Riabinin who with another classmate Oleg Kazakov taught math analysis in the Math School No. 91 from 1979 to 1982, and asked him the following questions:

1. Venia told me that he and Andrei would meet once a week with Konstantinov and would obtain new leaflets from him. They also discussed the classes and their teaching. They discussed and thought up new topics and new problems. How did you get the problem leaflets from Konstantinov? Did you (and/or Oleg) meet with him? Did he come to your class at School No. 91 while you were teaching? Did you and Konstantinov go on hikes with your math class and, if so, did you discuss your classes? If so, how?
2. You mentioned last time that you (and Oleg?) did not get enough help from Konstantinov. What kind of help? Specifically in what regard?
3. When you taught math analysis at School No. 91, did you study math yourself "for the soul"? Did you have some community at the time that also studied mathematics or not?
4. You mentioned last time that you gradually began to lose the fervor/interest/enthusiasm for teaching math. Why do you think this could have happened?

AR: I may have forgotten, but I can't recall any episodes of contacts with Konstantinov regarding that class—no leaflets, no hikes, no conversations. But I'm sure that if I had asked [Konstantinov] for help and I had been turned down, I would remember. … [I] think, I sensed, that I could not handle it. To do the job right, I had to give it much more time than I was prepared to. When this realization came, the enthusiasm disappeared. Then again, that's the way I reconstruct it now. At the time I was a different person whose emotions I don't remember. And I did little reflecting (like now, actually), so I can't remember my thoughts on this topic, either.

EM: My question is this: Looking back, what kind of help would you have wanted from Konstantinov?

AR: I think it would have been extremely useful if someone had talked to me seriously beforehand about what this work would require of me. Looking back, I understand that I did not recognize the responsibility I had taken on. If I had recognized it, I would have asked for help (for those leaflets, for advice) much more actively, and I'm sure I would have gotten it. But as it was, somehow it didn't occur to me either that it was common practice or that without it I wouldn't be up to the job.

EM: Wait a minute, wait a minute! You mean you wrote up the leaflets yourself with Oleg? You didn't use Konstantinov's?

AR: It's possible we used them at first, but I don't remember anything anymore. Given how young we were, it seemed that since we had already covered all of this, we should now be able to repeat it ourselves.

EM: I see. In other words, you and Oleg tried to recall, find or think up your own problems? Like that?

AR: Well, yes. … I now think that either Konstantinov either over relied on us, which would have been odd, or more likely he didn't expect anything beforehand, rightly hoping that Mironych [Vladimir Mironovich Sapozhnikov, then a regular math teacher at School No. 91 and Konstantinov's colleague] would get us through, one way or another.

EM: Perhaps. And did Mironych help?

AR: Nope. I barely knew him.

EM: It seems odd to me how much he helped Venia and Andrei (they met once a week) and how much he left you and Oleg to the mercy of fate. You had to invent a math class practically from scratch. Oy-yoy-yoy. . .

AR: Well, not from scratch—we had just been shown everything with us as a model [of being math school students at School No. 91 in 1974 – 1977].

EM: Yes, but the material that had been done and how to work with it and how to change it. It sounds like that wasn't available to you; nor was the pedagogical support that he gave Venia and Andrei. They discussed with him interesting solutions that we were doing. They thought up new problems with him, especially for us. They created a new topic. Yes, and they simply interacted every Thursday—that's not unimportant!

AR: That's what I'm saying—if I had recognized the responsibility, I would have asked for all that. Now I consider that work my failure, although the youngsters apparently thought well of me and think well now. They probably didn't especially expect much, either (Skype conversation between EM and AR, December 2, 2016).

When I asked a similar question to Alexander Shen who first taught a math class in School No. 91 in 1980 – 1983 (a cohort of year younger students than Alexei Riabinin), he replied that his experience with Konstantinov's guidance and support was between Venia and Alexei but closer to Alexei. "I consulted him [Konstantinov] about how to make sure that the kids didn't raise a ruckus. . . Well, it's well known how kids raise a ruckus—they would talk during lessons, they don't listen to what is being said to them, etc., the standard situation (nothing criminal). . . . And I remember his answer to this day: a book about animal trainers, said Konstantinov, states that if the trainer is sincerely convinced that the animals will obey him, then they will do so. . ." Alexander reported that he had not have dissatisfaction with support and guidance provided by Konstantinov.

Venia Dardyk hypothesized that Alexei Riabinin and Oleg Kazakov's experience with Konstantinov's lack of support might relate to their lack of initiative of asking Konstantinov for help. He elaborated that the latter could be due to Alexei's wrong reason for becoming a math teacher. As Alexei reported to me, and I shared with Venia, Alexei became a math teacher in 1979 because he thought he owed the community and Konstantinov and wanted to pay back with his volunteering. In contrast, Venia describes his own entry into teaching a math class differently. Venia says that he LOVED teaching math in Konstantinov's math circles, he loved developing new math problems, he loved listening to students' solutions, he loved testing these solutions for holes, he loved finding math beauty with the students, and he loved providing guidance when they were stuck. Venia tells that his teaching partner Andrei had the same reason for teaching. However, Venia also reported that he could relate to Alexei's teaching experience as well, because at the last half year of his teaching, he and Andrei had lost some enthusiasm for teaching math due to the changes in his overall interests away from math and instead had enjoyed interactions with his high school students (us) outside of math.

It seems to me that Konstantinov expected his math teachers to have wings in mathematics and teaching and actively seek help and support when they needed. That was not necessarily true. It does not seem that he was actively building a pedagogical community of his teachers with wings.

Part IV: Brief History of Konstantinov's Math Schools

LB: How did the system of math classes begin to proliferate? It sprang up at School No. 7, as you say, but how did the process run after that? Was it difficult to "wangle" permission?

NK: No, at that point it was easy to wangle, it was the beginning of the 1960s, but later all of it had to be defended. The point was that at the Nineteenth Congress of the ACP

(b) [All-Russia Communist Party (Bolsheviks), the Congress was held from October 5 – 14, 1952[37]]– after which it became the CPSU—Stalin put forth a very strange idea. First, that it was time in our country, on the road to communism, to abolish money, and second, we should transition to a direct exchange of products.

LB: In other words, go back far into the past?

NK: Yes, go back to prehistoric times. Anyway, when he came out with this, everyone politely kept quiet, except the first secretary of the Estonian Communist Party, Nikolai Karotamm. He began to ask Stalin how this would be done. Like a real Estonian, he approached the issue seriously, without a sense of humor. Stalin answered him: "Come to see me this evening." But how was he to get there? He had no plane, but he persuaded the commander of the military district to give him a military plane. Karotamm flew in to Moscow, and suddenly began to comprehend that he had done something wrong. This was the result. Stalin said to him: "I see you are interested in theoretical questions. You will work in Moscow, at the Institute of Economics, as a senior research associate." And he did not return to Estonia, the people there were simply at a loss, wondering "What happened to the first secretary?"

Also, at the Nineteenth Congress [of the Communist Party of the USSR], Stalin quoted Marx and said that the division of labor was a terrible stain on capitalism and that a person who gets an occupation is shackled to that occupation for life. Stalin recalled this remark by Marx, but again, no one understood what had to be done and how. A few years later, under Khrushchev, a man named Semichastny[38] emerged, who initially was, according to standard practice, the first secretary of the Komsomol, and then became chairman of the KGB. Later he was ousted—he had wanted too much power.

At some point he spoke at a party congress and repeated the idea that polytechnical education was needed. And, then, Khrushchev imposed percentage quotas, dictating how many people from workers' families should be accepted at higher educational institutions and declared that rabfaks [schools preparing workers for higher education] were needed, that workers should be pushed toward higher educational institutions. Meanwhile, an 11-year educational system was introduced in schools, and two days a week pupils had to do practical work in industry. As a result, Kronrod suddenly said: We will take advantage of this moment to create math classes. And we declared that there should be industrial specialization—"programming"—and we would teach mathematics. Two days a week in these classes were devoted only to math and programming. Kronrod was the head of the computational math lab of the Institute of Theoretical and Experimental Physics (ITEP), and I was a graduate student under him. I started this work partly prepared. I already had the "leaflets." I also had a program, which of course varied. There have been as many versions of programs as there have been years of math classes, the programs have always been authorial.

LB: And that is how math schools began to appear. When did School No. 2 appear?

NK: I'll tell you now how School No. 2 appeared. The first year we recruited some excellent youngsters, and everything was fine. But when we were recruiting for the second year, Volkov, the school principal, who was a member of the Oktyabr'skii [Moscow] Region Committee, was ordered to limit the recruitment of Jews. He shied away from saying this aloud. They could talk about it among themselves, but officially—no way. Volkov was too embarrassed to tell Kronrod about this and did the following: he did an inspection of the grade books of the schoolchildren who were applying. Whoever had a poor grade book was not accepted. Although there was an arrangement, whoever was added to the list after an interview was accepted. And he began to reject people, including the son of Izrail Moiseevich

Gel'fand. The boy actually applied to School No. 7 in 1963. The principal rejected him without realizing whom he was rejecting. And then Izrail Moiseevich immediately arranged the recruitment of a math class at School No. 2 (Borusiak, 2010).

NK: [Under the Khrushchev regime, 1956 – 1964] the stupid idea came up that schools should give kids industrial training. And then they established eleven grades instead of ten in school. That is when they added one year in school.

EM: Excuse me, the 11th grade was already after the 10th, in other words, kids graduated from school at age 18?

NK: Yes. And in the process the program was not expanded. They simply stretched out the same program over eleven years. So, then, Kronrod—do you know that name?

EM: I've heard it, I've heard it from you.

NK: Kronrod was an extraordinary person who accomplished something very important. And he somehow did this in a quiet way, so not many people now know that it was his handiwork. He said the following: "Now that an 11th grade has been added, we ("we" meaning mathematicians) have gained an opportunity, a unique one and the only one in the world, to establish math schools." And then he persuaded several mathematicians, including me, to teach math classes. That is when math classes first appeared.

EM: Interesting. Like industrial classes?

NK: Yes, like industrial classes. Programming had already been incorporated into the training there. However, the point was for mathematicians to be in charge there; not some unappealing teachers who don't know math, but mathematicians had to be in charge. They had to set the program, so that everything came from mathematicians. And so they persuaded several people, including me—I resisted, but they talked me into it—to teach math in those classes. So I started teaching math there, and when those kids graduated from the math school three years later, they had to think about where to go next for their studies, after they graduated from the math classes. And nearly everyone went to the mechanics and math department (Conversation between EM and NK, part 3, November 3, 2016).

LB: And how did the Kolmogorov boarding school come about?

NK: The first initiative came from Novosibirsk. The point was that those were years (1961, 1962, 1963) when there were a great many kids who were interested in math. They were youngsters born after the war, when there was a demographic explosion—a large generation.

LB: And probably the strong interest in physics that was typical of that period also played a role?

NK: Yes, and that interest in physics was very easy to explain. The test of the Soviet atomic weapon was conducted in September 1949. The Americans detected it, because American planes detected the radioactive clouds. But they couldn't understand from those clouds what had actually occurred: what the power was, whether it was a bomb or simply some device. They didn't know whether the Soviet Union had a bomb, they only knew that it had something.

The American administration—the president was Truman—had an idea: if the Soviet Union was preparing an atomic bomb, we should already now destroy all the enterprises for which we know their location. Later, when the Soviet Union acquires a bomb and the danger of a retaliatory strike arises, it will be too late. So if we are going to bomb, then do it now, in September 1949. But Truman could not bring himself to take that step, because they didn't know anything for sure: if the Soviet Union already had a bomb, it would be too dangerous. As a result, the Soviet government was compelled to encourage this project; however, where were the physicists to be found? Then, the recruitment for physics departments immediately doubled. The university was designed for 6,000 students, then their number increased to 30,000, in other words everything ballooned. Now they don't know how to reduce the number of students.

LB: However, was the Kolmogorov boarding school also supposed to recruit gifted children?

NK: Yes, yes. The Kolmogorov boarding school sprang up in the same burst as the Novosibirsk boarding school, but it was a little later than School No. 7. Just a year later.

LB: Then, very soon afterward, School No. 57 appeared.

NK: Yes, No. 57 appeared that way. Bogdanova, the head of the Frunzenskii [Moscow] Region Public Education Department, called me and said that they wanted to set up a math school.

LB: In other words, within a short period of time it became prestigious to have a math school, if every [Moscow] region wanted to have one. Were these schools that a region's front office could be proud of?

NK: Not every [Moscow] region, but that one did. I asked her: "Which school?" She responded, No. 57. I was familiar with that school, first, because a member of my Beta circle had sung its praises to me; he was a pupil there. And I realized that that was a good school and something could be organized there. Later I found out that my grandfather had graduated from the Carl Masing Secondary School and that is, in fact, School No. 57.

LB: Amazingly, there are always some coincidences popping up. And how did it turn out that these schools began to give specially selected, talented children not only strong math training but also tried to expand their social-sciences horizons, give them a strong cultural program and arrange hikes and trips? In fact, it's been like that from the very outset and is going on to this day.

NK: I will tell you about hikes. First, employees of Kronrod's lab came to us at School No. 7—that was one class. I was working in another one, with a good acquaintance of mine as well. But it turned out a couple of years later that mathematicians who were ready to work with school-children had disappeared, there were no more of them—

20 people were recruited, but no one else wanted to or was able to (Borusiak, 2010).

NK: Of course. One time there was a professor from Australia at one of our summer conferences, and he told us how their summer trips for schoolchildren differed from ours.

LB: They also have summer schools.

NK: Yes. There are summer schools in Europe, in the States. … He said they have two social groups: professors and schoolchildren. But you, he said, have three social groups, that is, undergraduates as well, and that's very important. There is always an invisible glass wall between professors and schoolchildren: no matter how they try to interact, they are going to end up interacting among themselves, in their own style. But the undergraduates break down this wall.

LB: Plus, it's simply impossible for a school to have as many instructors as are required for teaching math in specialized classes.

NK: Well, that's not the most important thing. I repeat, the appearance of undergraduates in school was a necessity, because of the shortage of teachers. At first it didn't exist, because Kronrod brought his whole lab. No one thought that undergraduates would be needed, there were plenty of adults.

LB: You mean that discovery was purely an accident?

NK: Yes, an accident (Borusiak, 2010).

Conclusions

In conclusion, I want to make three comments regarding my pursuit and investigation of Konstantinov's innovative authorial math pedagogy: philosophical, sociological, and methodological. My philosophical comment is regarding the purpose of education. Reading the aforementioned text, I can hear at least four types of goals attributed to education. The first is a goal of conventional education that teaches students important knowledge, skills, and attitudes that society preselects. Konstantinov is clearly against this goal, criticizing it for its deadliness. For Konstantinov, mathematics is a “living science, still intensively being created, and not a rigid body of knowledge that one must learn and then apply, as mathematics teachers often tend to believe and explain to their pupils” (Karp and Vogeli, 2010, p. 217). The curriculum has to be engaging for a student’s authorial agency here and now, which hardly make it preset by the society and preset for all. Additionally, I can add that rapidly and accelerating changes of the society make it difficult for society to predict what will be important in the future, generally, and for an individual student, specifically. My colleague Marjanovic-Shane and I described this goal of education as “alienated learning” (Matusov and Marjanovic-Shane, 2012).

The second purpose of education that I can sense in the text is authorial socialization of the students into a targeted practice (e.g., mathematics). It seems that early Konstantinov committed to this purpose and some of his colleagues are still committed now. Thus, Konstantinov and his colleagues Gerver and Kurshnerenko wrote in 1965:

> The problems presented here constitute a course in calculus. The collection contains the necessary definitions for independently solving all problems. By going over the material in this way, students master the techniques of mathematical thinking step by step. To master such techniques on a serious, professional level is the main aim of the course (cited in Karp and Vogeli, 2011, p. 292).

In my view, this quote reflects a transition of Konstantinov to the second goal of education where the emphasis is on independent problem solving but not creativity and authorship. Learning is still defined here technologically as mastery of mathematical thinking techniques. However, this tendency is already there. Here, education is viewed, first, as professional education. Later, a targeted practice of education becomes viewed as a creative process: “lived science” not reducible to knowledge and techniques. Similarly, learning this practice also becomes viewed as creative, unique, and authorial socialization. We called it “open socialization” (Matusov and Marjanovic-Shane, 2012).

Third, however, Konstantinov seems to move from the professional education as open, authorial socialization in a targeted practice (e.g., math) to education as promoting students' "wings" even though these wings—student's self-realization, self-actualization, creativity, self-expression—can be developed in different spheres rather than the targeted practice (e.g., math). The education for wings requires holistic, multi-subject, multi-practice, ontological, pedagogical support of students who may develop their interests and authorship in diverse, multiple, and changing spheres. Math can be one of such spheres among many others. The ideal educational environment for Konstantinov's pedagogy for wings seems to be a diversity and richness of students' authorial learning interests and pedagogical support for them. In our typology of educational goals, we did not envision this goal of education for wings (Matusov and Marjanovic-Shane, 2012).

The fourth goal of education lurking in the aforementioned texts is authorial critical examination of life, self, world, and practices. This goal of education goes back to Socrates who claimed that "the unexamined life is not worth living" (Plato, 1997, Apology, 38a5 – 6). The focus of many of Konstantinov's educators and students on critical thinking and "internally persuasive discourse" in math and other spheres is rather evident in his math schools. Karp and Vogeli described Konstantinov as an encyclopedic "freethinker" (2010, p. 210) and I would add also a critical thinker who was not limited to math or even sciences. This goal of education as authorial critical examination of life, self, world, society, and practices definitely shaped Konstantinov's pedagogy. Using Aristotelian terminology, we called this goal "praxis of praxis" (Matusov and Marjanovic-Shane, 2012). This diversity of educational goals definitely shaped Konstantinov's pedagogical practice and it has been a subject of open and hidden debates among his teachers and students.

My second comment is sociological. Although propelled by Khrushchev's political liberalization (1956 – 1964), Konstantinov's innovative pedagogy and practice of math circles, math schools, and math Olympics tournaments flourished during political, economic, social stagnation, and stability of the Brezhnev's regime (1964 – 1982/1986). There have been diverse cover-up political justifications that Konstantinov and his colleagues had to navigate in their struggle to promote and maintain their innovative educational practices and institutions: the concurrent Communist ideology, military concerns, nation-state prestige, and so on. His own justification as education for wings, opening and promoting creative potential in students, has remained underground. It seems that the societies of the second part of the twentieth century and the beginning of the twenty-first century—both Socialist and Capitalist—are not ready for this type of pedagogy. It can be because the modern society still remains essentially instrumental where people live to survive and support themselves rather than they survive and support in order to live. This type of the society views education also instrumentally as a servant to surviving and maintaining practices such as economy, military, political regime, and so on. Konstantinov's view of education is intrinsic and not instrumental. He views education as an inherent human existential lifelong need among other inherent human existential lifelong needs (e.g., art, love, science, dialogue). This need cannot be reduced to something else; it is the final cause in itself (i.e., education for education sake). Although education can serve other human spheres and practices, this function of education is secondary.

My American colleague Anne Morris, a math education professor, raised an interesting question, "When I imagine myself trying to implement such an approach in the U.S., I have to be aware that there are very different cultural factors that would affect students' eagerness and interest in self-motivated math activity. How does active and overt repression of the individual [in the Soviet Union and in modern Russia] affect their commitment to intellectual autonomy?" Would students without repression of the individual eager to volunteer in their own education? I think this is an empirical question to study. My hypothesis is that the freer people are from pressures of the necessity, the more eager are they involved in their own genuine education (see Matusov et al., 2017).

My last comment is methodological (although I dislike and critized this term; see, Matusov, 2017; Matusov and Brobst, 2013). I view Konstantinov's innovative pedagogical views and practice as alive and dialogic. Konstantinov has changed his pedagogical views and he experimented with his practice. His views are full of inconsistencies, contradictions, and controversies. Diverse participants, diverse authorships, and diverse circumstances have shaped Konstantinov's innovative pedagogical practice (I should, probably, use plural here but I want to consider the totality of his innovative pedagogical practices). It has been experienced and interpreted differently by diverse participants and observers. My goals here were to describe and engage in this diversity and to develop my own authorial vista and utterance on it to contribute to the Big Dialogue on education in general and on Konstantinov's innovative pedagogy in specific. I hope my utterance will generate many alternative utterances by others who may or may have not experienced Konstantinov's pedagogy for wings.

Notes

1. For Russian names not known in the West, I use the Library of Congress transliteration system https://www.loc.gov/rr/european/comintern/comintern-translit.html. For the known Russian names, I use their existing spelling.
2. https://en.wikipedia.org/wiki/Nikolay_Konstantinov
3. Alexander Poddiakov, an alumnus of No. 91 Math School, remembers that he also mathematized the world around of him. "For example, while waiting in a barber's shop, I was killing time by examining the reflection of a vertical electrical cord in a decorative mirror cylinder. I was thinking about a formula of the function that could describe the reflected cord. Would it be easier to find this formula, if I moved my perspective vista to the cylinder? I would not say that I constantly think about math but periodically an emerging math problem becomes a vortex that sucks me in. . .."
4. Ivan Turgenev was a famous Russian writer of the nineteenth century (https://en.wikipedia.org/wiki/Ivan_Turgenev).
5. In Russian, the gender of the pronoun refers to the gender of noun. In this case, the noun "person" has the male gender. Russian nouns have three gender: male, female, and neutral.
6. See https://en.wikipedia.org/wiki/Cantor%27s_diagonal_argument
7. Literally "self-publishing" in Russian – an illegal production of forbidden literature in the Soviet Union (https://en.wikipedia.org/wiki/Samizdat).
8. Alexander Shen, who administered entrance exams for the Math Schools Nos. 91 and 57 in 1977 and then taught in Math Schools Nos. 91 and 57, disagrees, "At least in the majority of classes of the 'Konstantinov system' that I saw, performance was considered above all, of course. Because overall there could have been several times more participants who even reached the end of the process

than there were places" (11/30/2016). It sounds to me like when there were too many candidates passing the last round, there was a selection by the number (and probably quality/difficulty) of solved problems. As a result, in this case, the selection of candidates seemed to be two steps: 1) by students persevering interest and 2) by their achievement on the exams. Alternatively, some of the people administrating the entrance exams to math schools might not share Konstantinov's educational philosophy. I wonder if a lottery or a combination of super high achievement and a lottery can be used in a case of too many candidates getting through the sex-round entrance exams. At the same time, the following is a description of a more resent selection to the Dubna Summer School on "Contemporary Mathematics," inspired by Konstantinov's pedagogy. "Most of the students are current or former Olympiad prizewinners, but some places are reserved for students without such distinctions, who apply to the school via the Internet. They are accepted if the organizers appreciate how they have filled in the school's rather unusual application forms, in which they are asked to describe, in brief essay form, their interest in mathematics, answering questions such as: What was the last mathematical book that you have read and how did you like it? or: What mathematical proofs are your favorites (present two)? or further: What mathematical constructions have most impressed you? The recommendations of teachers, especially teachers of selective schools, are also taken into account" (Karp and Vogeli, 2010, p. 216). Again, the focus seemed to be on the students' deep interest in math and not on abilities.

9. At that time, Soviet comprehensive school was 1 to 10 grades (from 7-year old to 17-year old), after which some students could join institutes (i.e., specialized colleges) and universities or go to professional schools or directly join work. Elementary, middle, and high school were in the same building sharing subject teachers. There were no electives. All classes were mandatory. The classes were organized by cohorts who moved together from class subject to class subject. The cohorts were often stable; therefore, in some Soviet schools, children stayed together from the 1st to 10th grade.

10. https://en.wikipedia.org/wiki/Intelligentsia

11. http://www.vzmsh.ru/

12. https://en.wikipedia.org/wiki/Racetrack_(game)

13. My other classmate Oleg Kazakov doubts that it was at the beginning of the 8th grade and thinks it is more likely to be at the beginning of the 9th grade.

14. I thought that scheduling of our math analysis classes was at the end of the school day by the teachers' design so we could freely leave the class and school without affecting our other classes. However, both Nikolay Konstantinov and my former math analysis teacher Venia Dardyk told me that scheduling math analysis classes at the end of a school day was incidental, based on Venia and Andrei's availability. There was no pedagogical design to give the students a choice to leave school earlier or solve math problems elsewhere at their desire.

15. https://en.wikipedia.org/wiki/Martin_Gardner

16. http://en.wikipedia.org/wiki/Continuous_function

17. http://tutorial.math.lamar.edu/Classes/CalcIII/StokesTheorem.aspx

18. https://en.wikipedia.org/wiki/Newton%27s_laws_of_motion#Newton.27s_1st_Law

19. Poddiakov was in a physics class and he remembers creating math-physics problems such as "What should a form of a vase be in order to have a constant proportion between the area of the surface of water and the water volume in the vase during the water's evaporation. I came to a beautiful answer involving the degree of x."

20. The Club of Amateur Song (CSR, KSP in Russian) was an informal social movement that emerged in the Soviet Union and united lovers of authorial bards' songs and singing. Mostly, the CSR abbreviation has been used as a synonym for bard songs and singing. CSR also refers to the broader subculture, the "Sixties," which also includes the Komunar movement, a number of tourist clubs, and so on (https://ru.wikipedia.org/wiki/%D0%9A%D0%BB%D1%83%D0%B1_%D1%81%D0%B0%D0%BC%D0%BE%D0%B4%D0%B5%D1%8F%D1%82%D0%B5%D0%BB%D1%8C%D0%BD%D0%BE%D0%B9_%D0%BF%D0%B5%D1%81%D0%BD%D0%B8 in Russian).

21. Lev Iosifovich Sobolev is a honored teacher of Russian and Russian literature and the director of a theater in Moscow http://gym1567.mskobr.ru/common_edu/shkol_noe_otdelenie_shkola_75/obwie_svedeniya/pedagogicheskij_kollektiv/kafedra_slovesnosti/sobolev_lev_iosifovich/

22. https://en.wikipedia.org/wiki/Decembrist_revolt

23. https://en.wikipedia.org/wiki/Pavel_Pestel

24. I contacted him and he could not remember such a reply on the survey. He suspects that his classmates confused him with somebody else because he has only positive memories of his school experiences. Unfortunately, Ira Gertseva lost her archive with the survey replies several years ago.

25. The emigration pathway for Jews was very difficult but opened in the 1970s until the beginning of the Soviet invasion of Afghanistan started in late December 1979. It reopened only in 1988 under Gorbachev, when I left the country.

26. https://en.wikipedia.org/wiki/Bella_Subbotovskaya

27. https://en.wikipedia.org/wiki/Bauman_Moscow_State_Technical_University

28. Popular-science mathematics and physics journal "*Kvant*" ("Quantum" in English, published since January 1970) http://kvant.mccme.ru/.

29. Sergey Dorichenko is the Editor-in-Chief of the *Kvantik* journal (http://kvantik.org/). Kvantik means "little Kvant" in Russian.

30. Aleksandr (Alexander) Semenovich Kronrod (Russian: Алекса́ндр Семёнович Кронро́д) (October 22, 1921 – October 6, 1986) was a Soviet mathematician and computer scientist, best known for the Gauss-Kronrod quadrature formula that he published in 1964 (https://en.wikipedia.org/wiki/Alexander_Kronrod).

31. Alexander Shen argues that it has not been a universal approach in all math schools, "In all the other classes of the 'Konstantinov schools' there were requirements of one kind or another, and pupils who did not fulfill them dropped out…" I wonder if there has been a disagreement or a lack of understanding between Konstantinov and some of his colleagues.

32. Hilbert's 13th problem was solved by Soviet mathematicians Kolmogorov and Arnold https://en.wikipedia.org/wiki/Hilbert's_thirteenth_problem

33. Pyotr L. Kapitsa was a Nobel Prize winner in physics, https://en.wikipedia.org/wiki/Pyotr_Kapitsa.
34. https://en.wikipedia.org/wiki/Andrey_Kolmogorov
35. https://en.wikipedia.org/wiki/Diophantine_equation
36. http://trv-science.ru/2017/04/25/mnogoobrazie-konstantinova/
37. https://en.wikipedia.org/wiki/19th_Congress_of_the_Communist_Party_of_the_Soviet_Union
38. Vladimir Smichastny, https://en.wikipedia.org/wiki/Vladimir_Semichastny

Acknowledgments

First, I want to thank Nikolai Nikolaevich Konstantinov for generously offering me his ideas, memories, reflections, and time. I want to thank Veniamin (Venia) Dardyk, Aleksei (Liosha) Riabinin, Irina (Ira) Kazakova (formerly Gertseva), Oleg Kazakov, Alexandra (Sasha) Shlyapentokh, Alexei (Liosha) Saverchenko, Leonid (Lionia) Rozenbaum, Evgenii (Zhenya) Emelin, and Yuliy (Yulik) Baryshnikov for interviews and answering my questions. I want to thank Alexander Shen for sharing Konstantinov's archive with me, discussing issues, and providing his critical feedback on an early version of the text; Sergey Dorichenko, Oleg Gritsenko, Pavel Yakushkin, Liubov' Borusiak, and Vladislav Kokushkin for helping me connect with Nikolai Nikolaevich and for their advice. I am also thankful for many Russian journalists and colleagues who interviewed Nikolai Nikolaevich and posted their interviews on the Internet. In addition, I want to thank my doctoral students in math education at the University of Delaware—Joseph DiNapoli, Siobahn Young, Tony Mixell, Jenifer Hummer, and Laura Willoughby—for their interest in the innovative math pedagogy and their challenging questions guiding my inquiries and interviews. Thanks a lot to Alexander Poddiakov for sharing his experiences as an alumnus of No. 91 Math School. I am especially thankful to Alexander Poddiakov, Anne Morris, and Leda Echevers, Ana Marjanovic-Shane, Mark Smith, and Jennifer Hummer for providing her feedback and editing an early version of the manuscript. Finally, I want to thank Pentti Hakkarainen, Mike Cole, and Irina Burns, the editors and staff of the JREEP, for supporting the project and Steven Shabad for his excellent translation of most of the Russian texts.

ORCID

Eugene Matusov http://orcid.org/0000-0001-7587-2266

References

Anderson, M.A. 2010. *Investigating the Impact of an Engaged Adolescent Participation in Hip-Hop Pedagogy: An Exploration of the Educational Possibilities of Youth Culture*. PhD Doctoral dissertation, University of Delaware, Newark.

Bakhtin, M.M. 1991. *The Dialogic Imagination: Four Essays by M. M. Bakhtin*, trans. C. Emerson and M. Holquist. Austin, TX: University of Texas Press.

Berlyand, I.E. 2009. “A Few Words about Bibler’s Dialogics: The School of the Dialogue of Cultures Conception and Curriculum.” *Journal of Russian & East European Psychology*, vol. 47, no. 1, pp. 20– 33. doi: http://dx.doi.org/10.2753/RPO1061-0405470101

Bibler, V.S. 2009. “The Foundations of the School of the Dialogue of Cultures Program.” *Journal of Russian & East European Psychology*, vol. 47, no. 1, pp. 34– 60. doi: http://dx.doi.org/10.2753/RPO1061-0405470102

Blum, S.D. 2016. *“I Love Learning; I Hate School”: An Anthropology of College*. London, UK: Cornell University Press.

Borusiak, L. 2010, September 29. “Kak poiavilos’ matematicheskoe obrazovanie: Beseda s matematikom Nikolaem Konstantinovym” [“How Did Math Education Emerge: Conversation with Mathematician Nikolai Konstantinov”]. Accessed from Polit.ru. http://www.polit.ru/article/2010/09/29/matheducation/

Denisenko, A. 2017. “Kolmogorovskii internat i Vtroraia shkola: Na dvukh poliusakh odnoj planety” [“Kolmogorov’s Boarding School and Math-School No 2: On the Two Poles of the Same Planet”]. *Sem’ Iskusstv [Seven Arts]*, vol. 82, no. 1. Accessed http://7iskusstv.com/2017/Nomer1/Denisenko1.php

Dewey, J. 1956. *The Child and the Curriculum and the School and Society* (Combined ed.). Chicago, IL: University of Chicago Press.

Dorichenko, S. 2010. “Interv’iu s N. N. Konstantinovym” [“Interview with N. N. Konstantinov”]. *Kvant*, no. 1, pp. 19 – 23. Accessed Ielementy bol’shoi nauki [Elements of the Big Science] http://elementy.ru/lib/431023

Falk, J.H., and L.D. Dierking. 2002. *Lessons without Limit: How Free-choice Learning is Transforming Education*. Walnut Creek, CA: AltaMira Press.

Falk, J.H., E. Donovan, and R. Woods. 2001. *Free-choice Science Education: How We Learn Science Outside of School*. New York, NY: Teachers College Press.

Frenkel, E. 2013a. “The Fifth Problem.” In *Love and Math: The Heart of Hidden Reality*, ed. E. Frenkel, 28– 38. New York, NY: Basic Books, a member of the Perseus Books Group.

Frenkel, E. 2013b. *Love and Math: The Heart of Hidden Reality*. New York, NY: Basic Books, a member of the Perseus Books Group.

Geertz, C. (1973). *The interpretation of cultures: Selected essays*. New York, NY: Basic Books.

Gubailovskii, V., and A. Kostinskii. (2004, June 2). Shkol’nye olimpiady po matematike [School math olympiads]. Accessed from Radio Svoboda [Radio Liberty]. http://www.svoboda.org/a/24197560.html

Ilyashenko, Y.S., and A.B. Sossinsky. (2010, March). “The Independent University of Moscow.” *The Newsletter of the European Mathematical Society*, vol. 75, pp. 38– 44. Accessed http://www.mccme.ru/mathinmoscow/img/IS.pdf

Imaikin, V. (1998). "N. N. Konstantinov o matematicheskom obrazovanii" ["N. N. Konstantinov about mathematic education"]. *Matematicheskoe Obrazovanie [Mathematic Education]*, vol. 2, no. 1. Accessed http://lomonosovclub.com/ru/study_lists/54#!/articles/295

Karp, A., and B.R. Vogeli. 2010. *Russian Mathematics Education: History and World Significance*. Hackensack, NJ: World Scientific.

Karp, A., and B.R. Vogeli. 2011. *Russian Mathematics Education: Programs and Practices*. Hackensack, NJ: World Scientific.

Konstantinov, N.N. 2001. Rossiiskie matematicheskie klassy [Russian math classes]. http://www.mccme.ru/teachers/articles/russmath.htm).

Konstantinov, N.N. 2007. "How People Guessed that Kepler's Laws are Consequences of the Mechanics Laws" [in Russian]. *Mathematical Eduction*, vol. 40, pp. 16– 25.

Konstantinov, N.N., V. Bugaenko, M. Vialogo, and S. Dorichenko. 2002. "Matematicheskie kruzhki ran'she…" ["Mathematic circles before…"]. *Matematicheskoe Prosveshchenie [Mathematic Enlightenment]*, vol. 3, no. 6, pp. 38– 48.

Latour, B. 1987. *Science in Action: How to Follow Scientists and Engineers through Society.* Cambridge, MA: Harvard University Press.

Lave, J. 1992. "Word Problems: A Microcosm of Theories of Learning." In *Context and Cognition: Ways of Learning and Knowing*, ed. P. Light and G. Butterworth, 74– 92. Hillsdale, NJ: Erlbaum.

Lave, J. 1992, April. "Learning as Participation in Communities of Practice." *Presented at the Meeting of the American Educational Research Association*. Accessed http://www.udel.edu/educ/whitson/897s05/files/Lave92.htm

Leenson, E. (2012, November 5). "Uchitelia bol'shogo goroda: Nikolai Konstantinov" ["Teachers of the Big City: Nikolai Konstantinov"] *Bol'shoi Gorod [The Big City]*. Accessed http://bg.ru/kids/uchitelja_bolshogo_goroda_nikolaj_konstantinov-15493/

Lobok, A. 2012. "The Writing Person." *Journal of Russian and Eastern European Psychology*, vol. 50, no. 6, pp. 32– 68.

Matusov, E. 2009. *Journey into Dialogic Pedagogy.* Hauppauge, NY: Nova Science.

Matusov, E. 2011. "Authorial Teaching and Learning." In *Bakhtinian Pedagogy: Opportunities and Challenges for Research, Policy and Practice in Education across the Globe*, ed. E. J. White and M. Peters, 21– 46. New York, NY: Peter Lang.

Matusov, E. 2015a. "Chronotopes in Education: Conventional and Dialogic." *Dialogic Pedagogy: An International Online Journal*, vol. 3, pp. A65– 97. doi: 10.5195/dpj.2015.107

Matusov, E. 2015b. "Legitimacy of Non-negotiable Imposition in Diverse Approaches to Education." *Dialogic Pedagogy: An International Online Journal*, vol. 3, pp. A174– 211. Accessed http://dpj.pitt.edu/ojs/index.php/dpj1/article/view/110/105

Matusov, E. 2017. "How to Engage Practitioners from across the Learning Landscape in the Research Enterprise: Proposal for Phronêtic Research on Education." *Integrative Psychological and Behavioral Science*, vol. 51, no. 1, pp. 94– 119. doi: 10.1007/s12124-016-9365-3

Matusov, E., D. Baker, Y. Fan, H.J. Choi, and R.L. Hampel. 2017. "Magic Learning Pill: Ontological and Instrumental Learning in Order to Speed Up Education." *Integrative Psychological and Behavioral Science*, *51*(3), 456–476. doi: 10.1007/s12124-017-9384-8

Matusov, E., and J. Brobst. 2013. *Radical Experiment in Dialogic Pedagogy in Higher Education and Its Centaur Failure: Chronotopic Analysis*. Hauppauge, NY: Nova Science.

Matusov, E., and A. Marjanovic-Shane. 2012. "Diverse Approaches to Education: Alienated Learning, Closed and Open Participatory Socialization, and Critical Dialogue." *Human Development*, vol. 55, no. 3, pp. 159– 66. doi: 10.1159/000339594

Matusov, E., and A. Marjanovic-Shane. 2017. “Promoting Students’ Ownership of Their Own Education through Critical Dialogue and Democratic Self-governance (Editorial).” *Dialogic Pedagogy: An International Online Journal*, vol. 5, pp. E1– 29. Accessed https://dpj.pitt.edu/ojs/index.php/dpj1/article/view/199. doi:10.5195/dpj.2017.199

Matusov, E., M.P. Smith, E. Soslau, A. Marjanovic-Shane, and K. von Duyke. 2016. “Dialogic Education from and for Authorial Agency.” *Dialogic Pedagogy: An International Online Journal*, vol. 4, pp. A162– A197. doi: 10.5195/dpj.2016.172

Matusov, E., and P. Sullivan. 2017. “Pedagogical Violence.” *Dialogic Pedagogy: An International Online Journal*.

Matusov, E., and K. von Duyke. 2010. “Bakhtin’s Notion of the Internally Persuasive Discourse in Education: Internal to What? (A Case of Discussion of Issues of Foul Language in Teacher Education).” In *Proceedings from the Second International Interdisciplinary Conference on Perspectives and Limits of Dialogism in Mikhail* Bakhtin Stockholm University, Sweden June 3–5, *2009*, ed. K. Junefelt and P. Nordin, pp. 174– 99. Stockholm: Stockholm University.

Morson, G.S. 2004. “The Process of Ideological Becoming.” In *Bakhtinian Perspectives on Language, Literacy, and Learning*, ed. A.F. Ball and S.W. Freedman, pp. 317– 31. Cambridge, UK/New York: Cambridge University Press.

Plato. 1997. *Complete Works*, trans. J.M. Cooper and D.S. Hutchinson. Indianapolis, IN: Hackett Pub.

Privalov, A. 2012, January 17. “Inkubator dlia Perelmanov” [“The Incubator for Perelmans”]. *Iekspert online [Expert online]*. Accessed http://expert.ru/2012/01/17/ugol-zreniya/

Shifman, M.A. 2005. *You Failed Your Math Test, Comrade Einstein: Adventures and Misadventures of Young Mathematicians or Test Your Skills in Almost Recreational Mathematics*. Hackensack, NJ: World Scientific.

Smirnov, 2004. Присутствуя при рождении (с точки зрения учителя) [Being present at the birth (from the point of view of a teacher)]. Второшкольная газета [*The Newspaper of School No 2*], 1. Accessed www.sch2.ru/gazeta/n1/smirnov.php; http://ilib.mccme.ru/2/biogr.htm

Szpiro, G.G. 2007. “Bella Abramovna Subbotovskaya and the ‘Jewish People’s University.’” *Notices of the AMs*, vol. 54, no. 10, pp. 1326– 30.

Appendix: Konstantinov’s Math Leaflet Samples

PART II. 1972 – 1973 Academic Year

Math Circles at MSU
(N.N. Konstantinov, supervisor)
Wednesday Math Circle
Leaflet No. 1

1. Given: a segment AB and a straight line that intersects the segment at some internal point. Find a point C on the straight line such that the line bisects the triangle ABC. What relative positions of the segment and the straight line result in more than one solution? Do such relative positions exist?

2. Let k be an integer. If we divide k^2 by 4, what remainder can result?
3. Is it possible to lay out all domino tiles following the rules of the game in such a sequence that there is a one at one end and a six at the other?
4. Prove that $k^3 - k$ is always divisible by 6 without a remainder (k is an integer).
5. Point A lies inside a circle. Find the locus of the centers of the chords of the circle that pass through point A.
6. Given: 80 gold coins. One of them is fake (lighter than the others). Using four weighing operations on a regular two-pan balance scale without weights, find the fake coin.
7. A self-intersecting pentagon having the form of a five-point star (not necessarily regular in shape) is considered. Find the sum of the angles at the ends of the star's "rays."
8. A fish tank initially contained 10 pike of various sizes. Then, their number diminished, since the pike were swallowing one another. Let us call a pike "sated" if it swallowed other pike twice. If a sated pike is eaten, it is still considered to be sated. What is the maximum number of sated pike that can result?

Introduction to Mathematical Analysis of Problems for the Course 9th Class

N.N. Konstantinov 1971

Leaflet 1 (AS)

Addition and Subtraction

A set D of real numbers contains the member 0, and addition and subtraction operations are defined in this set, i.e., a + b and a − b are determinate numbers if a and b are numbers.

Furthermore, the following basic properties (axioms) apply:

1. a + b = b + a
2. (a + b) + c = a + (b + c)
3. a + 0 = a
4. From a − b = c, it follows that b + c = a.

Comment. For all axioms, the wording "free variable letter" means that the axiom is a true statement for all possible values of this letter. If nothing particular is said about a letter then it is valid to assume that any real number can be represented by this letter. Exercises (derived inferences).

1. Prove that $((a + b) + c) + d = a + (b + (c + d))$

Comment. In a sum $a + b + c + d$, one does not need to add the brackets because the result does not depend on how the brackets are placed. The exact wording of this statement in a general form will be given in leaflet 9, where you will have the basis for the proof of this fact. In all leaflets, except the 1st and 9th it is allowed to use this fact as given without a need to prove it. Definition. $-a$ means $0 - a$, ($-a$ is called a number, which is the opposite to a).

2. $A + (-a) = 0$.
3. There is only one number that has the property of zero, such as there is only one number X [i.e., zero] such that for any number a, $a + X = a$.
4. For any numbers a and b, there is only one number X that $a + X = b$ (i.e., for given numbers a and b, there is only one number that has the property of their difference; if $b = 0$, this fact means that only one number has property that is the opposite number to a).
5. $- c = a + (- a)$.
6. Explain reducing the left and right sides of an equation by the same addend. Explain moving an addend to the other side of an equation with an opposite sign.
7. If the same number is added to both sides of an incorrect equation, the result will be an incorrect equation.
8. $- - a = a$.
9. Prove that $-(a + b - c + d - e)= -a - b + c - d + e$ (the expression $a + b - c + d - e$ should be understood as follows: $(((a + b) - c) + d) - e)$. This is how the rule of signs when opening parentheses is explained. Hereafter it may be used without proving it.

Matusov, E. (2017). Nikolai N. Konstantinov's authorial math pedagogy for people with wings: Special issue. *Journal of Russian & East European Psychology, 54*(1), 1-117. doi: https://doi.org/10.1080/10610405.2017.1352391

Table 1. List of errors and corrections

Page	**Wrong text appeared in the journal**	**Corrected text**
6	"and later received a Ph.D. in physics"	"and later received a Ph.D. in mathematics"
13	"Finally, when a_n is 9, then the original number is 473684210526315789, and n = 17."	"Finally, when a_n is 9, then the original number is 947368421052631578, and n = 17."
13	"If a_n is 6, then the original number becomes infinite (636842) 1578947368421052, and, thus, should be rejected as a possible solution."	"If a_n is 6, then the original number is 631578947368421052, and n = 17."
13	"There are only seven possible answers (of course, each number can be "doubled," "tripled"—e.g., 210526315789473684210526315789473684—and so on but I was not interested in it). Additionally, noticed that the numbers create a rotating pattern of the same digits in the same order, except for a_n = 6."	"There are only eight possible answers (of course, each number can be "doubled," "tripled"—e.g., 210526315789473684210526315789473684—and so on but I was not interested in it). Additionally, noticed that the numbers create a rotating pattern of the same digits in the same order."
14	"Again, I presented this elegant solution at our math facultative. It generated many interesting discussions, including new math problems of why the numbers rotated and why a_n = 6 was an exception. Unfortunately, I do not remember if we solved these new problems or not."	"Again, I presented this elegant solution at our math facultative. It generated many interesting discussions, including new math problems of why the numbers rotated. Unfortunately, I do not remember if we solved these new problems or not."
17	"Georg Cantor that points on a part of a lane cannot be"	"Georg Cantor that points on a part of a line cannot be"
28	"I → D at c □ I means that for every ε > 0 there exists a δ > 0 such that for all x □ I."	"I → D at c ∈ I means that for every ε > 0 there exists a δ > 0 such that for all x ∈ I."
32	"normative algorithm"	"normal algorithm"
32	"such as a -> b as this will replace all letters "a" in a given word to letters "b". For example, the word "appla" will become "bpplb"."	"such as a -> b as this will replace one letter "a" in a given word to letters "b". For example, the word "appla" will become "bppla"."
32	"NAURI"	"NAIRI"
33	"some of my classmates also remember inventing new gf.math problems but some did not."	"some of my classmates also remember inventing new math problems but some did not."
46		The first paragraph on the page has to be indented as it the continuation of the quote from p.45.
61	"Interviewer: But circles for the 6th grade are something different. It's not the same as a 6th-grader going to a 10^{th} grade circle."	"Interviewer: But circles for the 6th grade are something different. It's not the same as a 6th-grader going to a 10th grade circle."
96	"(like Alexander Shen and Sergey Dorichenko, 1980 – 1983)"	"(like Alexander Shen, 1977-1980, and Sergey Dorichenko, 1980 – 1983)"
106	Kurshnerenko	Kushnirenko
110	"getting through the sex-round entrance exams"	"getting through the six-round entrance exams"